Measuring spatial chirp in ultrashort pulses using single-shot Frequency-Resolved Optical Gating

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Abstract: We show that the spatio-temporal distortion, spatial chirp, is naturally and easily measured by single-shot versions of second-harmonic generation frequency-resolved optical gating (SHG FROG) (including the extremely simple version, GRENOUILLE)`. While SHG FROG traces are ordinarily symmetrical, a pulse with spatial chirp yields a trace with a shear that is approximately twice the pulse spatial chirp. As a result, the trace shear unambiguously reveals both the magnitude and sign of the pulse spatial chirp. The effects of spatial chirp can then be removed from the trace and the intensity and phase vs. time also retrieved, yielding a full description of the spatially chirped pulse in space and time.

OCIS codes: (320.7080) Ultrafast devices, (320.7100) Ultrafast measurements

References and links

1. P. O'Shea, M. Kimmel, X. Gu, and R. Trebino, "Highly simplified ultrashort pulse measurement," Opt. Lett., **26**, 932-934 (2001).
2. R. Trebino, *Frequency-Resolved Optical Gating: The Measurement of Ultrashort Laser Pulses* (Kluwer Academic Publishers, 2002).
3. A. G. Kostenbauder, "Ray-Pulse Matrices: A Rational Treatment for Dispersive Optical Systems," IEEE JQE **26**, 1148-1157 (1990).
4. B. S. Prade, J. M. Schins, E. T. J. Nibbering, M. A. Franco and A. Mysyrowicz, "A simple method for the determination of the intensity and phase of ultrashort optical pulses" Opt. Comm., **113**, 79-84 (1994).
5. C. Iaconis, C., I.A. Walmsley , "Self-referencing spectral interferometry for measuring ultrashort optical pulses", JQE **35**, 501-509 (1999).
6. C. Dorrer, E.M. Kosik, I.A. Walmsley, "Spatio-temporal characterization of the electric field of ultrashort optical pulses using two-dimensional shearing interferometry", Appl. Phys. B **74** [Suppl.], S209-S217 (2002)

1. Introduction

Because their generation involves considerable spatio-temporal manipulations, ultrashort laser pulses commonly suffer from spatio-temporal distortions. Probably the most common such distortion is *spatial chirp*, in which the average wavelength of the pulse varies spatially across the beam. Devices such as pulse compressors (see Fig. 1), which are standard in essentially all ultrafast lasers and apparatuses, deliberately introduce massive amounts of spatial chirp, only to—in principle—remove it afterward. After two prisms, the beam lacks angular dispersion, but has considerable linear spatial chirp. While the next two prisms of a pulse compressor, in principle, remove this effect, in practice they typically do not completely do so unless aligned perfectly. One cause of this distortion is that the first and last prism separations may not be

Received December 03, 2002; Revised January 10, 2003

equal. Using only two prisms and a mirror or mirrors to reflect the beam back on itself guarantees that the relevant prism separations are equal, but there are other causes of spatial chirp in pulse compressors even in such a simple two-prism arrangement: the beam may be diverging or converging while inside the device, or the prisms may be arranged at slightly different angles. As a result, the beam emerging from a pulse compressor is frequently contaminated with spatial chirp.

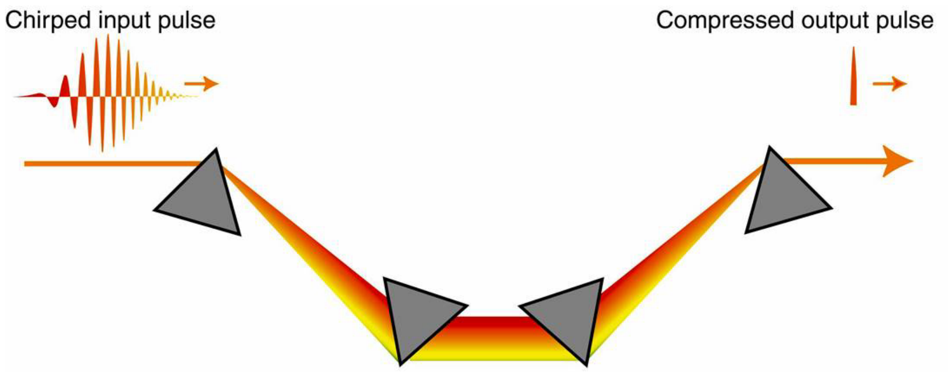

Fig. 1. A prism compressor, which utilizes four identical Brewster prisms (or two and a mirror), which, if misaligned, yields spatial chirp in a pulse. If the prism separations, apex angles, or incidence angles are not precisely the same, spatial chirp results. Even slight amounts of beam divergence or expansion inside this device can yield significant spatial chirp in the output pulse.

Worse, spatial chirp has many additional causes, including even optics that would seem beyond suspicion. For example, a window with a slight wedge, as is required for laser output couplers (to avoid feedback from the back surface), causes angular dispersion, which also imparts spatial chirp in the beam, and the further that the beam propagates from the optic the more spatial chirp. This is especially problematic in the most broadband (that is, the shortest) pulses. In addition, even a simple plane-parallel window yields unavoidable spatial chirp when it is tilted (Fig. 2). Thus, simply placing a (usually 45-degree) pick-off mirror in the beam causes spatial chirp in the transmitted beam.

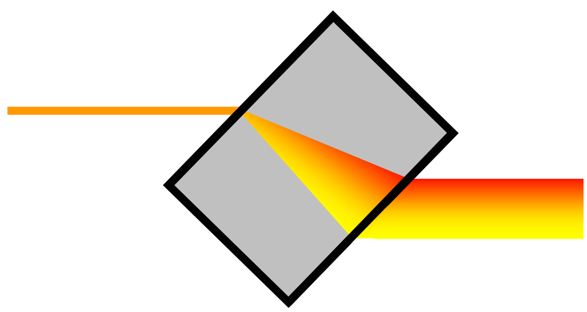

Fig. 2. An ultrashort pulse propagating through a simple plane-parallel window. Even a slight tilt of the window yields spatial chirp in the transmitted pulse, despite the absence of angular dispersion.

If a pulse has spatial chirp, experiments performed with it will yield inappropriate results. For example, each individual ray along the beam will contain only a fraction of the full pulse spectrum, and hence won't be as short as would be possible if the pulse possessed the full spectrum of the beam. Also, spectroscopic experiments performed with spatially chirped pulses will involve both exciting and probing with spatially varying wavelength, which could easily confuse their interpretation. Even worse are the potential effects of spatial chirp on a laser-induced-grating experiment. If the grating is induced with a spatially chirped pulse and its spatially reflected replica (i.e., *a pulse that has experienced, for example, one more or one less reflection*), it will be a stationary grating (as expected) in the beam center, but a *moving grating* at the edges due to the different center wavelengths of the two beams creating the grating in these regions. The moving grating will wash out due to its motion, in addition to excited-state decay. Such a grating will appear shorter-lived than might otherwise be imagined.

There is not a convenient diagnostic for spatial chirp. A spatially resolved spectral measurement, in principle, suffices, but aberrations in spectrometers can mimic this effect, so such measurements are not routinely made. Researchers have also used spatially resolved spectral interferometry [4] and spatially resolved SPIDER [5,6], but these interferometric methods are difficult to align and to keep aligned. SPIDER is also experimentally very complex and has within its apparatus a pulse stretcher, which significantly disperses the beam and requires very careful alignment or it will introduce spatial chirp itself. Also, spectral interferometry requires high stability of the absolute phase of the pulse to be measured. While the latter two methods have measured the full intensity and phase vs. one spatial co-ordinate (not just the spatial chirp), it is important to develop a device for measuring spatial chirp in ultrashort laser pulses that is simple, easy to use, reliable, artifact-free, and accurate.

In this note, we report such a device. Remarkably, it is a familiar one: any single-shot second-harmonic-generation frequency-resolved-optical-gating (SHG FROG) device, including the extremely simple SHG FROG device we recently reported—GRENOUILLE [1]. We will show that, *without a single modification*, single-shot SHG FROG and GRENOUILLE yield the pulse spatial chirp—in addition to the intensity and phase vs. time. Specifically, the ordinarily symmetrical SHG FROG trace develops an asymmetrical *shear* (tilt) in the presence of spatial chirp, which is proportional to the spatial chirp.

Even better, the inversion formula is very simple. First note that single-shot SHG FROG maps delay onto position and hence yields a plot of intensity vs. frequency and position, and a spatio-spectral diagnostic for spatial chirp involves a similar plot. The spatio-spectral plot develops a shear in the presence of spatial chirp. And so does the FROG trace. Indeed, we find that a spatially chirped pulse yields a single-shot FROG or GRENOUILLE trace with a shear that is approximately *twice* that of the spatial chirp when plotted vs. frequency and *one half* when plotted vs. wavelength.

This technique also works for higher (odd) orders of spatial chirp. And we show that the effects of spatial chirp may also be removed from the FROG trace, and the pulse intensity and phase can be determined in the usual manner. The retrieved intensity and phase may then be modified taking into account the spatial chirp, and a spatio-temporal measurement of the pulse obtained for a spatially chirped pulse.

2. Theory of spatial chirp in single-shot SHG FROG measurements, such as GRENOUILLE

To see the effect of spatial chirp on single-shot FROG measurements (see Fig. 3), we begin with the usual expression for an SHG FROG trace, including the carrier frequencies of the two pulses [2]:

$$I_{FROG}^{SHG}(\omega,\tau) = \left| \int_{-\infty}^{\infty} \{E(t)\exp[i\omega_0 t]\} \quad \{E(t-\tau)\exp[i\omega_0(t-\tau)]\} \quad \exp[-i\omega t] \; dt \right|^2 \quad (1)$$

which can be simplified to yield:

$$I_{FROG}^{SHG}(\omega,\tau) = \left| \int_{-\infty}^{\infty} E(t)\; E(t-\tau)\, \exp[-i(\omega - 2\omega_0)t]\, dt \right|^2 \quad (2)$$

In single-shot FROG techniques, two replicas of the pulse are crossed at a large angle, and delay is mapped onto position, $\tau = \alpha x$, where $\alpha = 2\ \sin(\theta/2)/c$. This yields: $I_{FROG}^{SHG}(\omega,\alpha x)$.

Now if we allow the pulses to have spatial chirp in a single-shot SHG FROG set up, we must replace ω_0 with a spatially dependent frequency: $\omega(x) = \omega_0 + \xi x$. Then the SHG FROG trace becomes:

$$I_{FROG}^{SHG\ sp\ ch}(\omega,\tau) = \left| \int_{-\infty}^{\infty} E(t)\exp[i(\omega_0+\xi x)t]\, E(t-\tau)\exp[i(\omega_0+\xi x)(t-\tau)]\exp[-i\omega t]\, dt \right|^2 \tag{3}$$

Simplifying this expression, we obtain:

$$I_{FROG}^{SHG\ sp\ ch}(\omega,\tau) = \left| \int_{-\infty}^{\infty} E(t)\, E(t-\tau)\exp[-i(\omega-2\omega_0-2\xi x)t]\, dt \right|^2 \tag{4}$$

which can be written in terms of $I_{FROG}^{SHG}(\omega,\tau)$:

$$I_{FROG}^{SHG\ sp\ ch}(\omega,\tau) = I_{FROG}^{SHG}(\omega-2\xi x,\tau) \tag{5}$$

Since, in single-shot FROG techniques, delay is mapped onto position, $\tau = \alpha x$, the single-shot SHG FROG trace of a pulse with spatial chirp will be:

$$I_{FROG}^{SHG\ sp\ ch}(\omega,\tau) = I_{FROG}^{SHG}(\omega-2\xi x,\alpha x) \tag{6}$$

This expression shows that the SHG FROG trace, which is normally symmetrical with respect to delay, $I_{FROG}^{SHG}(\omega,-\alpha x) = I_{FROG}^{SHG}(\omega,\alpha x)$, develops shear in the presence of spatial chirp and no longer exhibits such symmetry. Because no other effect is known to cause such asymmetry, this is a simple and clear indicator of spatial chirp.

GRENOUILLE is a type of single-shot FROG measurement, but it (like single-shot methods that involve mirrors inserted halfway into the beam) involves *spatially* splitting the beam in two, rather than splitting the beam with a beam splitter. In other words, the left side of the beam gates the right side of the beam, rather than the entire beam gating itself. However, it yields the same slope (Fig. 4), so the result for GRENOUILLE is identical.

At this point, the observant reader might note that, since GRENOUILLE uses a Fresnel biprism to split and cross the beams, it would seem that it also introduces spatial chirp into the beams, which might perhaps bias the measurement. However, not only is this spatial chirp very small in magnitude (since the apex angle is very close to 180^o), but this spatial chirp is imparted with opposite sign onto the two beams inside the GRENOUILLE, and it cancels out of the analysis. There is also a very small amount of pulse-front tilt imposed by the biprism, but this is also taken into account by the standard delay calibration methods and does not affect the measurement.

Note that the above derivation also holds for all odd (i.e., higher) orders of spatial chirp. On the other hand, even orders of spatial chirp will produce symmetrical distortions in the trace and would be confused for pulse distortions in time and hence will require another (yet-to-be-invented) technique for their identification. Currently, the linear component of spatial chirp is of greatest interest (higher-order terms are generally very small), so henceforth we take “spatial chirp” to mean “linear spatial chirp.”

Note also that this result is independent of the pulse intensity and phase; the method is general.

#1905 - $15.00 US
Received December 03, 2002; Revised January 10, 2003

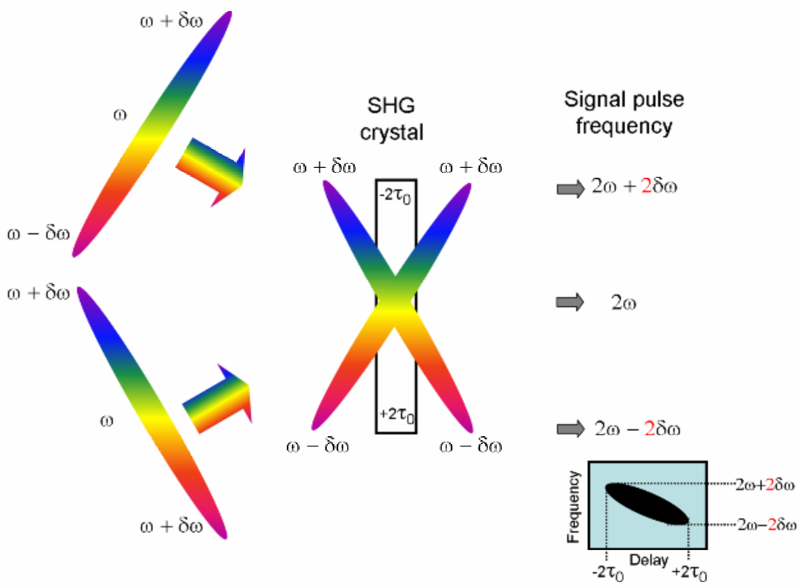

Fig. 3. Spatial chirp in single-shot SHG FROG. Two spatially chirped pulses are crossed at an angle in the SHG crystal. This yields variable delay mapped onto transverse axis. The crystal yields the autocorrelation signal of the pulse for the purpose of measuring its intensity and phase vs. time. However, spatial chirp causes a variation of the autocorrelation signal wavelength vs. distance (i.e., vs. delay). This yields a shear in the SHG FROG trace proportional to the magnitude of the spatial chirp.

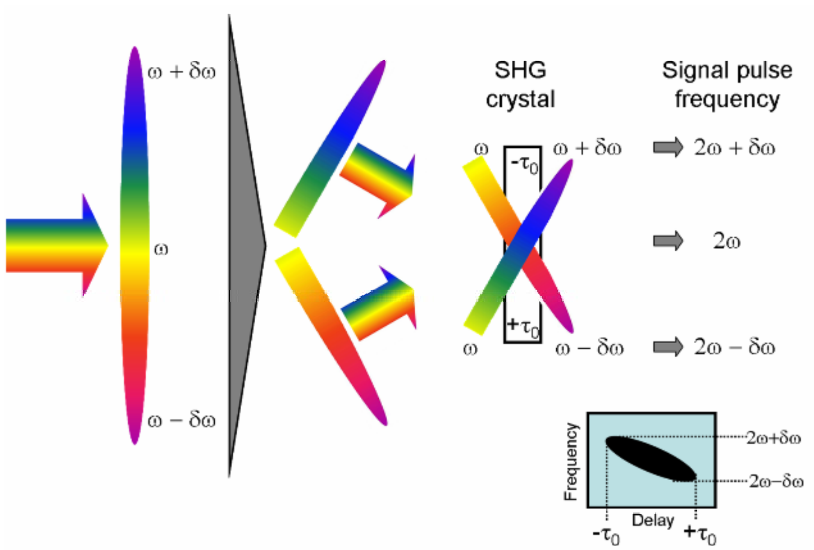

Fig. 4. Spatial chirp and GRENOUILLE. A spatially chirped pulse enters the Fresnel biprism from the left. The Fresnel biprism splits the pulse into two, which then cross in the SHG crystal. While the crystal yields the autocorrelation signal of the pulse for the purpose of measuring its intensity and phase vs. time, spatial chirp causes a variation of the autocorrelation signal wavelength vs. distance. This yields a shear in the GRENOUILLE trace proportional to the magnitude of the spatial chirp. Note that the slopes in both single-shot SHG FROG and GRENOUILLE are exactly the same.

3. Trace Shears in Single-Shot SHG FROG, GRENOUILLE, and Spatio-Spectral Plots

Measuring the spectrum vs. one spatial co-ordinate for a pulse yields a spatio-spectral plot. If the pulse has spatial chirp, $\omega(x) = \omega_0 + \xi x$, this plot will be sheared with slope ξ. This is the most obvious way to measure the spatial chirp, and it works well provided that the spectrometer is aberration-free.

Now, we can also compute the slope of the SHG FROG (or GRENOUILLE) trace vs. position. (We usually describe FROG and GRENOUILLE measurements in terms of the delay, but single-shot measurements map delay onto position, and position is the more natural unit for discussions of spatial chirp.) Simple examination of the expression for the sheared SHG FROG trace of a pulse with spatial chirp [Eq. (8)] shows that its frequency vs. position shear is $\omega_{ave}(x) = 2\xi x$. However, the position 'x' here is not beam transverse coordinate as in the case of spatio-spectral plots, but is instead the *crystal* transverse coordinate. They are simply related by a factor of $\cos(\theta/2)$, where θ is the beam crossing angle. So, the SHG FROG trace slope is $2\xi/\cos(\theta/2)$.

Since the cosine factor is approximately unity, the spatial-chirp-induced slope of the FROG trace is approximately *twice* the spatial chirp and twice that of the spatio-spectral trace when plotted vs. frequency. When plotted vs. wavelength, recall that the SHG FROG trace occurs at the second harmonic. Converting from frequency to wavelength, a factor of λ^2 must be included, reducing the slope of the FROG trace by a factor of 4 and yielding a new ratio of ½, rather than 2, for traces plotted vs. wavelength.

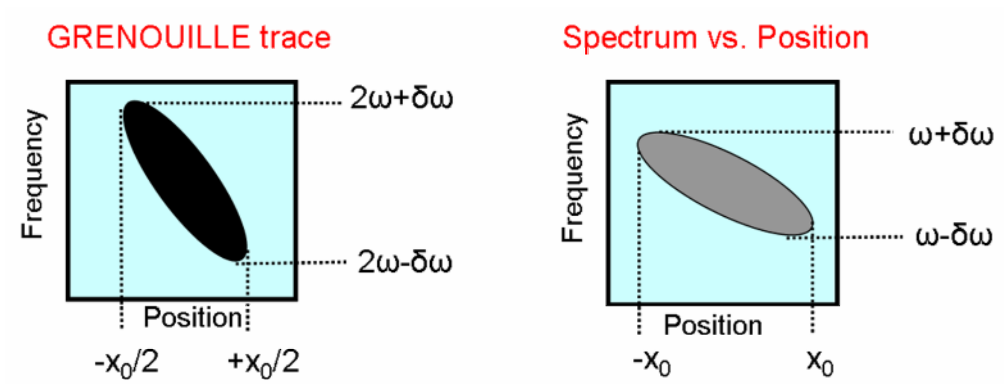

Fig. 5. Spatial chirp causes shear in both GRENOUILLE traces and spatio-spectral plots. The spatial-chirp-induced slope of the FROG trace is approximately *twice* the spatial chirp and hence twice that of the spatio-spectral trace.

4. Extracting the spatial chirp and intensity and phase from a linearly sheared trace

Finding the linear slope of the trace yields 2ξ. This can be done in several ways, but simply finding the difference between the centers of mass of the trace at +x and –x and plotting this result vs. x suffices to yield 2ξ. The spatial chirp can then be removed from the trace, and the true SHG FROG trace for E(t) is simply:

$$I_{FROG}^{SHG}(\omega,\alpha x) = I_{FROG}^{SHG\,sp\,ch}(\omega + 2\xi x, \alpha x) \tag{7}$$

The resulting trace is now the best estimate for the actual trace—and hence the pulse—in the absence of spatial chirp. The SHG FROG algorithm can then be run on the now symmetrical trace, yielding the pulse intensity and phase in the absence of spatial chirp. The spatial chirp can then be added back into the retrieved pulse, reproducing the pulse with the appropriate amount of spatial chirp. Note that the pulse will typically be longer when it is

#1905 - $15.00 US
Received December 03, 2002; Revised January 10, 2003

spatially chirped because its bandwidth along a given ray will be smaller due to the dispersing of the frequencies into different spatial regions, and this result will accurately reveal this fact.

The pulse can then be reconstructed using the retrieved intensity and phase and including the measured spatial chirp. If the FROG algorithm returns an intensity, I(t), and phase, $\phi(t)$, then the spatially chirped pulse field will be given by:

$$E(x,t) = \sqrt{I(t)}\exp[i(\omega_0 + \xi x)t - i\phi(t)]$$

where ξ is the measured spatial chirp. Note that the assumption of linear spatial chirp implies that the intensity and phase are independent of spatial co-ordinate. This would not be the case for *nonlinear* spatial chirp, in which the center frequency varies nonlinearly with position and the outer regions of the beam would necessarily have narrower spectra. But this result is exact for linear spatial chirp, the vast majority of cases, and we believe that it represents a significant improvement in practical pulse measurement.

5. Experiment

To introduce variable amounts of spatial chirp into a pulse, we modified the usual prism pulse compressor, placing mirrors between last two prisms, deflecting the pulse to two additional mirrors mounted on translation stage (see Fig. 6). By translating the latter two mirrors, we were able to align and misalign the compressor, obtaining positive, zero, or negative spatial chirp. Also, we aligned the compressor so that the angular dispersion was zero in all of our measurements, although we do not believe that the presence of angular dispersion would alter our results.

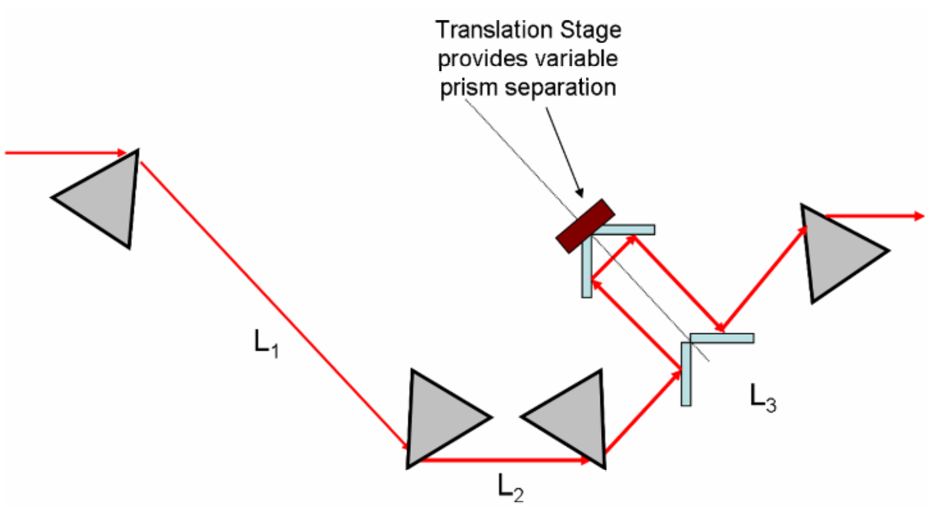

Fig. 6. Modified prism pulse compressor: translation stage used between last two prism provides variable prism separation and can be used to align and misalign the compressor.

We performed pulse measurements for various amounts of spatial chirp using GRENOUILLE. We determined the spatial-chirp parameter, ξ, from the measured GRENOUILLE trace from the linear slope of the trace (wavelength vs. delay) using the approach described in the previous section. We also made independent measurements of the spatial chirp parameter, ξ, from a spatially resolved spectral measurement using a carefully aligned imaging spectrometer.

Figure 7 shows GRENOUILLE traces and spatio-spectral plots of pulses with different amounts of spatial chirp for some of the experiments we have performed. The spatial chirp parameter ξ, on top of the figures was calculated from the details of our apparatus using

Received December 03, 2002; Revised January 10, 2003

Kostenbauder [3] matrices. The calculated numbers are rough (since they required knowledge of the exact path difference of the beams, which is not well defined when prisms are involved), but the calculations are of little importance here, as we have made independent measurements of the spatial chirp using the spatio-spectral plots. In any case, these figures nicely illustrate the effect of spatial chirp on experimental GRENOUILLE traces.

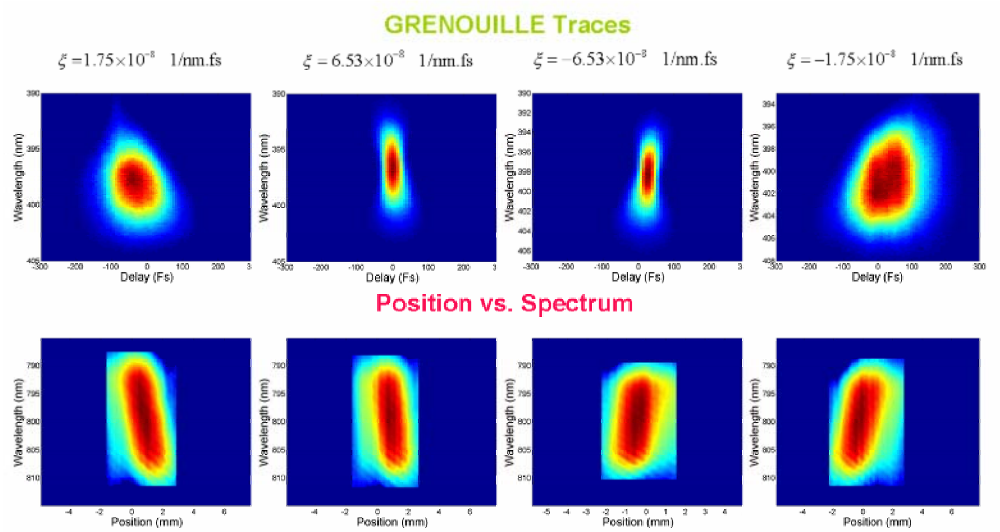

Fig. 7. Experimental GRENOUILLE traces and spatio-spectral plots. The shear in GRENOUILLE traces clearly reveals the existence and sign of spatial chirp.

The shear in the GRENOUILLE trace can be computed in many ways. The method we have used is to perform a Gaussian fit to the intensity vs. frequency slice in the trace for each position, and then finding the peaks at multiple positions and fitting to a line. We calculated the slopes of both the GRENOUILLE traces and spatio-spectral plots (Fig. 8), and we find that the slope of this plot, that is, the ratio of the GRENOUILLE trace slope and the spatio-spectral plot slope, is 0.49 +/- .027. This measurement agrees very nicely with the theoretical value of $\frac{1}{2}\cos(\theta/2) = 0.4995$ (the beam crossing angle is $\theta = 0.093$ radians for our experiment).

We find that GRENOUILLE can measure spatial chirp with high sensitivity. Using this method, we were able to align our prism pulse compressor with a sensitivity (in prism separation) of 0.4 mm. With such accuracy, this device should provide a practical and reliable alignment of pulse compressors used in ultrafast laser laboratories.

Received December 03, 2002; Revised January 10, 2003

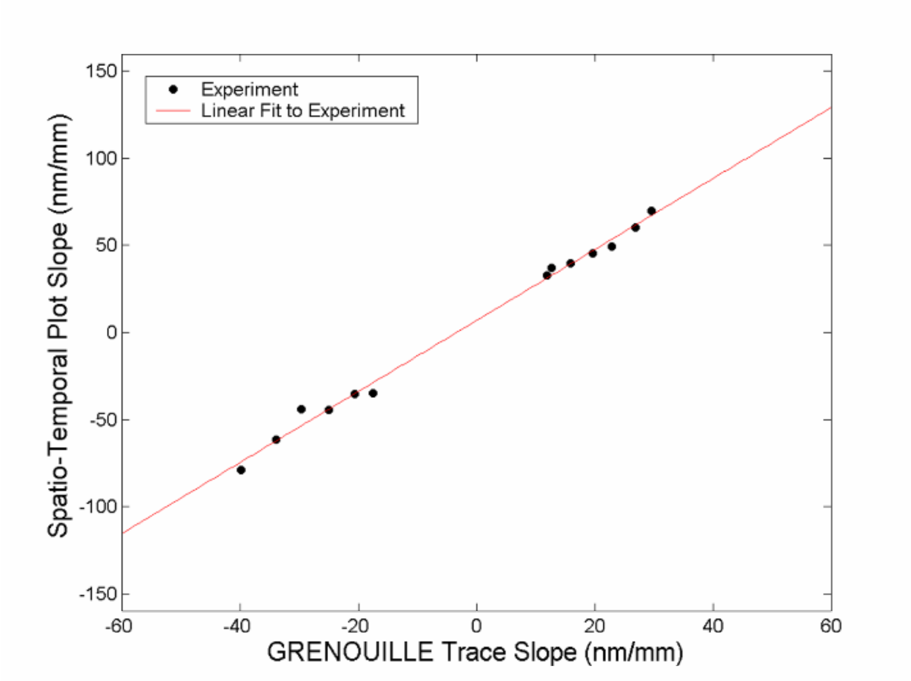

Fig. 8. Slopes of GRENOUILLE traces and corresponding spectrum vs. position slopes for various amounts of spatial chirp.

6. Retrieval of Pulse in the Presence of Spatial Chirp

We also performed a preliminary test of our approach for determining the full spatio-temporal intensity and phase vs. time and position for a pulse with linear spatial chirp.

We first retrieved the intensity and phase of a pulse without spatial chirp (i.e. without trace shear) from a properly aligned pulse compressor. We then misaligned the compressor, creating a spatially chirped pulse. We measured this pulse's (sheared) GRENOUILLE trace and then removed the shear from the originally sheared trace and retrieved the pulse from this new trace (see Fig. 10). This pulse was slightly longer and less broadband than the pulse we obtain when we aligned our pulse compressor for zero spatial chirp (see Fig. 9), as expected since the spatially chirped pulse should have less bandwidth along any given ray.

#1905 - $15.00 US
Received December 03, 2002; Revised January 10, 2003

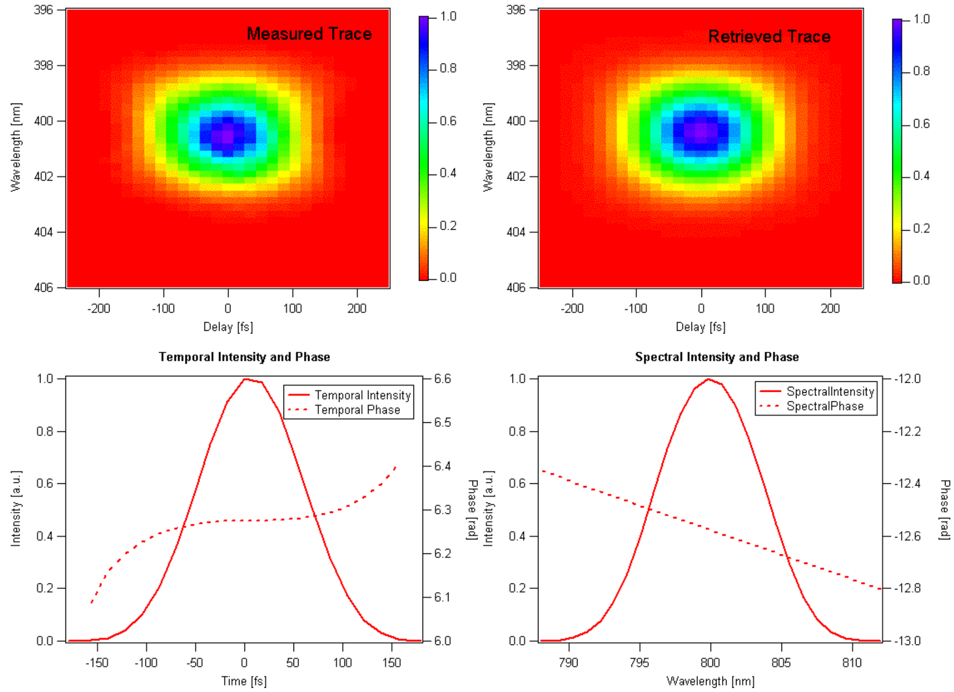

Fig. 9. Retrieval of intensity and phase of a pulse which does *not* have significant amount of spatial chirp. The FWHM pulse width is 123.7 fs. FROG error is 0.42% for this measurement.

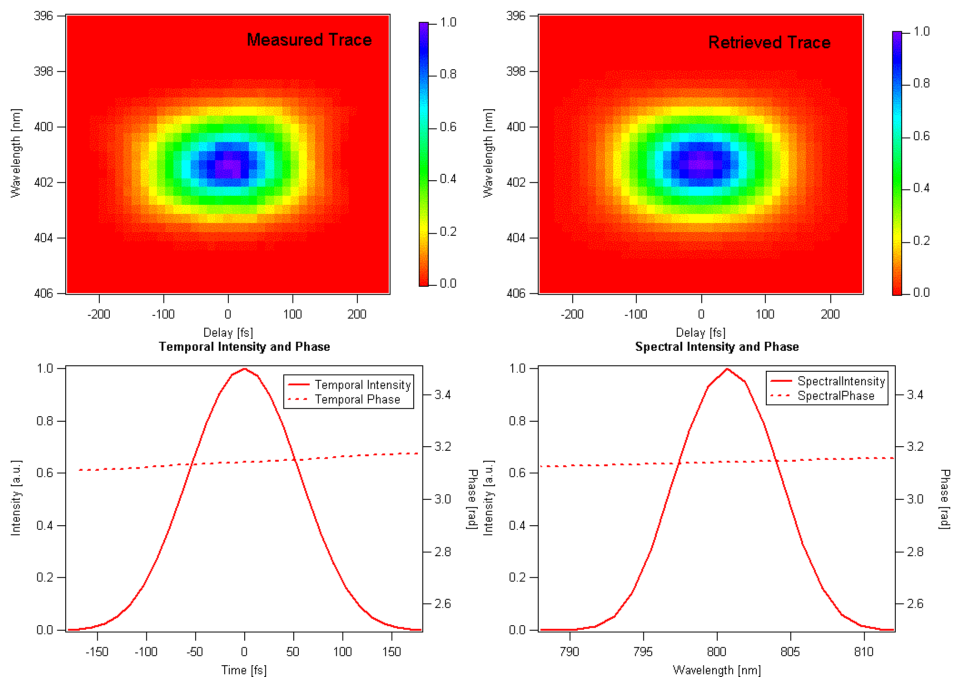

Fig. 10. Retrieval of intensity and phase of a pulse with spatial chirp after the shear is taken out from the trace. The FWHM pulse width is 129.3 fs. Note that the pulse broadens due to its narrower spectrum. The FROG error is 0.41% for this measurement.

#1905 - $15.00 US
Received December 03, 2002; Revised January 10, 2003

7. Conclusion

In conclusion, we have theoretically and experimentally demonstrated that single-shot SHG FROG and GRENOUILLE measurements not only yield the pulse intensity and phase, but the trace shear also sensitively yields the pulse spatial chirp. The shear in single-shot SHG FROG and GRENOUILLE is approximately twice the spatial chirp. In particular, we believe that GRENOUILLE's simplicity makes it an ideal diagnostic, not only for the pulse intensity and phase, but also for the spatial chirp.

Acknowledgements

This work was supported by National Science Foundation and a start-up grant from the Georgia Institute of Technology.