Induced-grating autocorrelation of ultrashort pulses in a slowly responding medium

Alfred M. Levine and Ercüment Özizmir

The College of Staten Island, City University of New York, Staten Island, New York 10301

Rick Trebino and Carl C. Hayden

Combustion Research Facility, Sandia National Laboratories, Livermore, California 94551

Anthony M. Johnson and Kathleen L. Tokuda

AT&T Bell Laboratories, Holmdel, New Jersey 07733

Received October 7, 1993; revised manuscript received April 5, 1994

We consider induced-grating autocorrelation (IGA) in a slowly responding medium and study three possible geometries (two-beam coupling, three-beam induced grating, and self-diffraction) in two different limiting cases (single-pulse experiments and many-pulse accumulated-grating experiments). We find that in five of these six cases the IGA trace is given by the squared amplitude of the electric-field correlation function, thus yielding information about the spectrum of the pulse. Theoretical expressions for the IGA trace are derived for both linearly chirped and self-phase-modulated pulses. Experiments performed with self-phase-modulated pulses are in excellent agreement with the theory. In this case we show how the measured IGA trace can be used to determine both pulse duration and pulse bandwidth.

1. INTRODUCTION

The measurement of the characteristics of an ultrashort pulse is perhaps the most significant and immediate problem facing the researcher in possession of one. As a result this problem has received tremendous attention over the past two decades. Early on it was recognized that the only tool available with sufficient temporal resolution to yield any useful information was the pulse itself. Of course the pulse cannot resolve itself, and using the pulse to measure itself yields only the pulse autocorrelation. Nevertheless, autocorrelation methods have been the only methods available for many years, and as a result they have become commonplace.

Initial work concentrated on intensity autocorrelation, and two-photon processes, such as two-photon fluorescence and second-harmonic generation (SHG), provided good results.[1,2] These two-photon methods and their relatives, such as two-photon absorption,[3] continue to be the standard methods used in ultrafast laboratories today.

Other nonlinear processes have been suggested and used for intensity autocorrelation,[4–6] but, with the realization that the most common type of distortion in femtosecond pulses is chirp, efforts in the past few years have shifted to developing methods for measuring the phase distortions common to ultrashort pulses. The pulse spectrum is not a bad indicator of the magnitude (but is bad for the sign) of linear chirp when the spectrum is accompanied by some measure of the pulse length, such as the intensity autocorrelation, but most researchers prefer to perform all measurements in the time domain because the required intensity autocorrelator is already an inherently time-domain device. Fortunately both types of information are provided in the time domain in a single device by the method developed by Diels and co-workers,[7–10] interferometric SHG autocorrelation (ISHGA), which involves performing SHG with a pulse that has propagated through a Michelson interferometer. This technique simultaneously yields the intensity autocorrelation and provides sufficient phase information to distinguish among transform-limited pulses, linearly chirped pulses, and pulses that have experienced simple self-phase modulation. It has been used to measure pulses as short as 6 fs, and, by placing a piece of glass in one arm of the Michelson interferometer, Diels and co-workers were able to distinguish the sign of the chirp of a pulse[11] as well as its magnitude. In addition, algorithms have been presented for the determination of the full pulse intensity and phase for simple pulses based on ISHGA in conjunction with one or two other experimental traces, such as the second-harmonic spectrum or interferogram.[12,13]

Additional methods have recently been developed that more directly provide phase information. Several techniques directly determine the instantaneous frequency versus time or its frequency-domain analog, the group delay versus frequency.[14–22] Chilla and Martinez[23–25] demonstrated a technique for directly determining the phase in the frequency domain and, using the easily measured pulse spectrum, extracted the approximate full pulse intensity and phase of a train of pulses. More recently, Kane and Trebino[26–28] showed that the pulse-characterization problem can be made equivalent to the well-known, solved problem of phase retrieval in two dimensions. With this knowledge, they demonstrated a class of techniques called frequency-resolved optical gating, which rigorously yields the full pulse intensity

0740-3224/94/091609-10$06.00

and phase evolution. This class of methods has the additional advantages of single-shot operation, simple apparatus, automatic phase matching, UV-through-IR wavelength range, high accuracy, and a robust retrieval algorithm.[29] Frequency-resolved optical gating can use any instantaneous nonlinear-optical process, but three-photon processes, such as the electronic Kerr effect, have been shown to function best.

Three-photon processes were used previously for autocorrelation measurements also.[30–53] We call this class of autocorrelation techniques induced-grating autocorrelation (IGA). (They are also called four-wave-mixing autocorrelation; for simplicity we exclude sum-generation processes in this discussion.) IGA has by no means been made obsolete by intensity-and-phase techniques. Intensity-and-phase methods all require a nonlinear medium with an instantaneous nonlinear response and as a result have not been demonstrated for extremely weak pulse trains such as those generated by mode-locked diode lasers for potential communications applications. IGA measurements, on the other hand, easily generate measurable signal strengths for such low-power pulse trains because they can take advantage of slow, and hence strong, nonlinear effects. Pulse trains of ~1-fJ pulses yield sufficient signal to generate a usable IGA trace in photorefractive media. Also, while IGA does not yield full intensity-and-phase information, it can yield at least as much information as is available from ISHGA.[30,33] IGA with an instantaneously responding medium (IGAFast) yields the (third-order) intensity autocorrelation, which has the advantage over two-photon autocorrelation methods of indicating pulse asymmetry. IGA with a slowly responding medium (IGASlow), on the other hand, yields the same phase information as in ISHGA.[30–35] Unlike with SHG, most materials exhibit $\chi^{(3)}$ effects and are potential samples, and UV operation is straightforward. Automatically phase-matched or nearly phase-matched beam geometries are readily available, simplifying alignment considerably and yielding quite large signal strengths. Also, because frequency-degenerate processes are the rule, group-velocity dispersion is significantly less of a problem than in SHG. In addition, the touchy alignment of the Michelson interferometer required in ISHGA is not necessary in IGA setups. Finally, whereas ISHGA has been achieved on a single-shot basis by use of a clever arrangement,[54,55] a single-shot IGA apparatus can be constructed in a more straightforward manner.[53]

IGAFast has been demonstrated experimentally with the phase-conjugate, polarization-gate, and self-diffraction beam geometries.[37–40] The theoretical treatment of IGAFast is also well established: the third-order intensity autocorrelation is a relatively straightforward concept and has the advantage that it can diagnose asymmetrical pulses. Numerous reports of experimental demonstrations of IGASlow in a wide range of samples (for example, dyes, photorefractive crystals, and thin amorphous films) also exist.[32–36,46–48,50–53,56] Theoretical investigation of IGASlow lags somewhat behind experimental efforts, however. It was only recently, for example, that commonly observed oscillations in the wings of IGASlow traces found an explanation.[33] Previous studies have untangled only some of the phase information available from this method.[49,56] Several years ago IGASlow with stochastic pulses was considered.[30] A couple of years ago we gave a brief description of IGASlow traces for a few specific types of commonly encountered deterministic ultrashort pulse but lacked the space to provide more than minimal quantitative information.[33] A full catalog of pulse types and their IGASlow traces is not yet available. Unfortunately IGA is complicated by the fact that, unlike second-harmonic-generation autocorrelation (SHGA), it permits several qualitatively different beam geometries and regimes. For example, the signal beam in an IGA experiment can either copropagate (and add coherently) with an input beam or not. In addition, in IGASlow one of the beams that forms the grating can also simultaneously probe it, or a separate beam can arrive later to do the probing independently of the excitation beams. There are slow media and very slow media (defined below): thus the problem can be further divided into single-pulse experiments, in which the only contribution to the grating occurs on a single laser shot, and multiple-pulse accumulated-grating experiments, in which the grating accumulates over many shots. In the first case the grating decay time is less than the time between laser pulses, and in the second case the opposite is true.

In this paper we discuss in detail IGASlow in both slowly and very slowly responding media. We consider two possible two-beam geometries (self-diffraction and two-beam coupling; see Fig. 1) and one three-beam geometry (e.g., phase conjugation; see Fig. 1) and perform calculations for the cases that have not been considered previously. We review some of the previously performed calculations for perspective. We find that, in all but single-pulse self-diffraction, the experimental trace can be adequately described by the same result, the squared magnitude of the pulse spectrum (in the deterministic limit). We then consider several specific types of ultrashort pulse: transform-limited pulses, linearly chirped pulses, and purely self-phase-modulated pulses. We show IGASlow traces for a range of values of the relevant parameters and compare these traces with those obtained from ISHGA. We find that (except in single-pulse self-diffraction) IGASlow yields traces that are exactly the magnitude-squared envelope of the high-frequency fringes of the two-beam interferogram. We also find that IGA yields essentially the magnitude-squared envelope of the high-frequency fringes of the trace obtained in ISHGA traces. The lack of high-frequency fringes in IGASlow is a significant advantage because many fewer data need to be taken. This permits, for example, straightforward single-shot operation, in which delay is transformed into a position on a multielement detector, which typically lacks sufficient elements to fully resolve an interferogram or ISHGA trace.

In the remainder of this paper we discuss only IGASlow. Consequently, we drop the Slow suffix except when the two versions of IGA must be compared.

2. GENERAL DESCRIPTION OF INDUCED-GRATING AUTOCORRELATION

The basic idea behind IGA is the splitting of a pulse and the interference in some medium of the resulting two ex-

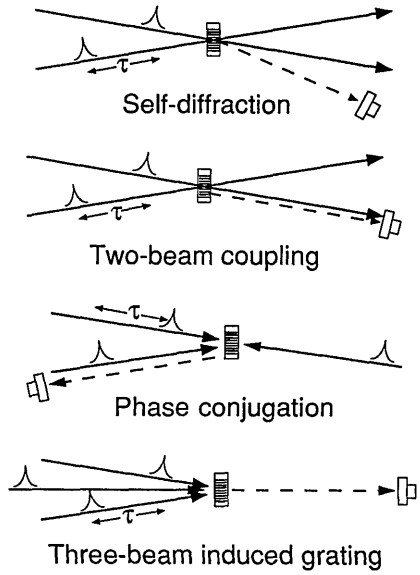


Fig. 1. Various beam geometries for performing IGA. Both three-beam geometries are essentially equivalent for this purpose.

citation pulses. The resulting interference fringes may then induce a modulation of the medium absorption coefficient or refractive index, i.e., a grating. This modulation is then probed by a third pulse or by one of the excitation pulses, and the diffracted light is detected. An IGA trace is produced by varying the delay between the two excitation pulses and plotting the diffracted intensity versus the relative delay. Because no interference fringes are obtained in the medium when the excitation pulses do not overlap in time, no grating can form, and no light can be diffracted. As a result it is clear that some properties of the pulse will be obtained in this manner. The property obtained will generally not be the pulse length, however, because, although it is necessary that the excitation pulses overlap for a grating to form, it is not sufficient. To understand this see Fig. 2, where we have illustrated the case of a linearly chirped pulse that has been split into two pulses, which then interfere in a medium with some delay between them. Notice that, at any point in time in the medium, unlike colors interfere. As a result the interference fringes will not be stationary. If the medium responds instantaneously, then the medium can follow these fringes no matter what their phase velocity; a relatively high diffraction efficiency will result. If the medium responds slowly, however, it will not be able to follow the fringes, and the material grating will wash out. In this case no light will be diffracted. Note also that if the delay between the excitation pulses had been zero then like colors would have interfered, and a strong grating would have resulted. Thus IGASlow will give information on phase distortions, such as chirp, in the pulse, but IGAFast will not.

IGASlow may be performed in a variety of media, based on a variety of effects, ranging from thermally induced refractive-index changes, to the optical Kerr effect, to the photorefractive effect, provided that the decay time of the grating in the medium is long compared with the pulse length to be measured and with any delays used in the interaction. We assume that the rise time is instantaneous, although many of the results of this paper hold even when this is not the case.

Figure 1 illustrates three types of beam geometry available for IGA experiments. The first is two-beam coupling, in which a pulse is split into two and interfered in the IGA medium. Here, the probe beam is the same as an excitation beam, and the signal beam copropagates and adds coherently with the other excitation beam. In two-beam coupling the power transfer between the two beams is measured. Another two-beam geometry is self-diffraction. Here the probe beam is also one of the excitation beams, but the signal beam propagates in a unique direction. Self-diffraction is not phase-matched, but if the angle between the beams is small then phase mismatch can be neglected. Three-beam geometries, such as phase-conjugation and all-forward-propagating arrangements, involve the probe's arriving separately and later, to probe the grating well after the excitation pulses but before the grating decays. The difference between these two three-beam geometries, shown in Fig. 1, is simply that the probe-beam directions are backward or forward. This difference does not enter into the analysis, so these two geometries are henceforth considered equivalent in every way.

We define τ_p as the pulse length, τ_d as the grating decay time, and T as the time between pulses in the train and distinguish between slow media and very slow media. Slow media integrate the grating over one pulse ($\tau_p \ll \tau_d \ll T$), probing the grating either during its formation or immediately afterward, whereas very slow media integrate the grating over many pulses ($\tau_p \ll T \ll \tau_d$). This distinction is important in one case (self-diffraction) but, interestingly, is inconsequential in all other cases.

3. THEORY

Let $E(t)$ be the electric field of the pulse to be measured. Also, let τ be the delay between the two replicas of the pulse that interfere in the medium. We assume that the dephasing time of the medium is much less than the other time scales in the problem, so that we may neglect coherence effects in the medium. We also assume that the rise time of the grating is fast compared with the

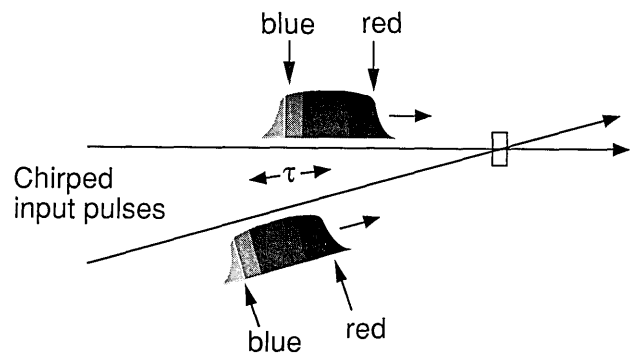


Fig. 2. Why pulses can overlap in a slowly responding medium and not yield a grating. If a chirped pulse is split and recombined in the medium with nonzero relative delay, then unlike colors overlap at all times. The fringe pattern sweeps through the medium, washing out the grating, which is the time integral of the fringes throughout the pulse. Thus IGASlow yields phase information.

pulse length and that the decay time of the grating is long compared with the pulse length and all delays between pulses. The grating amplitude is then given by

$$A(t, \tau) \propto \int_{-\infty}^{t} E(t')E^*(t' + \tau)\mathrm{d}t'. \quad (1)$$

The diffracted field is given by

$$E_{\mathrm{diff}}(t, \tau) \propto A(t, \tau)E_{\mathrm{pr}}(t), \quad (2)$$

where the probe field $E_{\mathrm{pr}}(t)$ can be $E(t)$ (as in self-diffraction), $E(t + \tau)$ (as in two-beam coupling), or another field (as in three-beam geometries).

We show that, for single-pulse two-beam coupling, single-pulse three-beam coupling, and multipulse experiments in all geometries, the detected signal is proportional to the squared magnitude of the electric-field autocorrelation function $k(\tau)$, defined by

$$k(\tau) = \int_{-\infty}^{+\infty} E(t')E^*(t' + \tau)\mathrm{d}t'. \quad (3)$$

We note that, in the limit as $t \to \infty$, the grating amplitude $A(t, \tau)$ is proportional to $k(\tau)$.

A. Three-Beam Geometries

We begin by considering three-beam arrangements in which the probe pulse arrives at the sample many pulse lengths after the two excitation pulses. In addition the diffracted pulse propagates in a unique direction, so the detected energy will be

$$W_{\mathrm{det}}(\tau) \propto \int_{-\infty}^{+\infty} |E_{\mathrm{diff}}(t, \tau)|^2\mathrm{d}t. \quad (4)$$

Substitution for $E_{\mathrm{diff}}(t)$ yields

$$W_{\mathrm{det}}(\tau) \propto \int_{-\infty}^{+\infty} \left| E_{\mathrm{pr}}(t) \int_{-\infty}^{+\infty} E(t')E^*(t' + \tau)\mathrm{d}t' \right|^2 \mathrm{d}t, \quad (5)$$

where we have taken advantage of the tardiness of the probe pulse to allow the upper limit of the t' integration to go to $+\infty$. Rearranging and factoring out the t-independent integral yields

$$W_{\mathrm{det}}(\tau) \propto \left| \int_{-\infty}^{+\infty} E(t')E^*(t' + \tau)\mathrm{d}t' \right|^2 \int_{-\infty}^{+\infty} |E_{\mathrm{pr}}(t)|^2\mathrm{d}t. \quad (6)$$

Now, the t integral is just the pulse energy of the probe pulse, so it can be lumped into the proportionality constant, yielding

$$W_{\mathrm{det}}(\tau) \propto |k(\tau)|^2. \quad (7)$$

This result is the squared magnitude of the Fourier transform of the intensity spectrum.

B. Two-Beam Coupling

In two-beam-coupling experiments the diffracted field copropagates with the other pulse, so the detected field should be written as

$$E_{\mathrm{det}}(t, \tau) \propto E(t) + E_{\mathrm{diff}}(t, \tau). \quad (8)$$

The detected energy is then

$$W_{\mathrm{det}}(\tau) \propto \int_{-\infty}^{+\infty} \{|E(t)|^2 + 2\,\mathrm{Re}[E^*(t')E_{\mathrm{diff}}(t, \tau)] + |E_{\mathrm{diff}}(t, \tau)|^2\}\mathrm{d}t. \quad (9)$$

In two-beam-coupling experiments one measures the change in pulse energy. Consequently the first term in the integrand must be subtracted out. Also, because we operate in the weak-signal limit, $|E(t)| \gg |E_{\mathrm{diff}}(t, \tau)|$, the last term may be neglected. Substituting for $E_{\mathrm{diff}}(t, \tau)$, we have

$$W_{\mathrm{det}}(\tau) \propto 2\,\mathrm{Re}\left[\int_{-\infty}^{+\infty} E^*(t)E(t + \tau) \times \int_{-\infty}^{t} E(t')E^*(t' + \tau)\mathrm{d}t'\mathrm{d}t \right]. \quad (10)$$

The t integration may be done by parts when it is observed that the t' integral is the complex conjugate of the antiderivative of the remainder of the integrand of the t integral. We can then rewrite this integral as

$$\int_{-\infty}^{+\infty} E^*(t)E(t + \tau) \int_{-\infty}^{t} E(t')E^*(t' + \tau)\mathrm{d}t'\mathrm{d}t = \left[\left| \int_{-\infty}^{t} E(t')E^*(t' + \tau)\mathrm{d}t' \right|^2 \right]_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} E(t)E^*(t + \tau) \int_{-\infty}^{t} E^*(t')E(t' + \tau)\mathrm{d}t'\mathrm{d}t. \quad (11)$$

But now the second term on the right-hand side of this equation is just the complex conjugate of the left-hand side. Combining these two terms on the left, we have

$$2\,\mathrm{Re}\left[\int_{-\infty}^{+\infty} E^*(t)E(t + \tau) \int_{-\infty}^{t} E(t')E^*(t' + \tau)\mathrm{d}t'\mathrm{d}t \right] = \left| \int_{-\infty}^{t} E(t')E^*(t' + \tau)\mathrm{d}t' \right|^2, \quad (12)$$

the left-hand side of which is precisely the desired quantity. Thus we have

$$W_{\mathrm{det}}(\tau) \propto \left| \int_{-\infty}^{+\infty} E(t')E^*(t' + \tau)\mathrm{d}t' \right|^2, \quad (13)$$

where we have substituted $+\infty$ into the upper limit of the integral. This, remarkably, is exactly the same result as in the three-beam case!

C. Self-Diffraction

In self-diffraction the diffracted beam propagates in a unique direction, but one of the excitation beams also acts as the probe beam, $E_{pr}(t) = E(t)$, so that the grating is being probed as it is created. Thus we have

$$W_{det}(\tau) \propto \left| \int_{-\infty}^{+\infty} E(t) \int_{-\infty}^{t} E(t')E^*(t' + \tau)dt'dt \right|^2, \quad (14)$$

which cannot be simplified further. Since we are interested in measuring the electric-field autocorrelation function, the single-shot self-diffraction geometry is not suitable for our purpose, and it will not be discussed further in this paper.

D. Multiple-Pulse Experiments

For the case of very slow media, for which the pulse repetition time T is much less than the grating decay time τ_d (i.e., $T \ll \tau_d$), the grating builds up over many pulses. Assuming that the pulse repetition time is much greater than the pulse width τ_p (i.e., $\tau_p \ll T$) and that the grating decay is described by an exponential, the grating amplitude is given by

$$A(t, \tau) \propto \int_{-\infty}^{t} E_0(t')E_0^*(t' + \tau)dt' + \sum_{j=1}^{\infty} \exp\left(-\frac{jT}{\tau_d}\right) \int_{-\infty}^{+\infty} E_j(t')E_j^*(t' + \tau)dt, \quad (15)$$

where $E_j(t)$ represents the jth previous pulse. Since the current pulse $E_0(t)$ is only a small part of this sum, we may neglect its contribution compared with that of the approximately τ_d/T other pulses, obtaining

$$A(t, \tau) \propto \sum_{j=0}^{\infty} \exp\left(-\frac{jT}{\tau_d}\right) \int_{-\infty}^{+\infty} E_j(t')E_j^*(t' + \tau)dt'. \quad (16)$$

When we assume that the pulse train contains identical pulses, this becomes

$$A(t, \tau) \propto k(\tau) \sum_{j=0}^{\infty} \exp\left(-\frac{jT}{\tau_d}\right). \quad (17)$$

Defining $N = \tau_d/T$ as the number of pulses during one decay time, we find that

$$A(t, \tau) \propto k(\tau) \frac{1}{1 - \exp\left(-\frac{1}{N}\right)} \approx Nk(\tau), \quad (18)$$

where the final expression is valid for large N. Thus the effect of multiple pulses is simply to multiply the grating strength by N. It is important to realize that this result is true for all geometries, including self-diffraction.

In the three-beam geometry the detector signal is proportional to $|A(\tau)|^2$ and hence to N^2. This is also true for self-diffraction. For the case of two-beam coupling, however, the detected field is $E_{det} \propto E(t) + A(\tau)E(t + \tau)$. In this case the detected signal intensity will be proportional to N instead of to N^2. For most experiments only the shape of the IGA signal is important, and this proportionality constant does not matter. There may be, however, some experimental situations in which distinguishing between N and N^2 will yield interesting information.

4. INDUCED-GRATING AUTOCORRELATION TRACES FOR SPECIFIC PULSE TYPES

In Section 3 it was shown that IGA traces are proportional to $|k(\tau)|^2$. In this section we calculate $k(\tau)$ for three types of pulse: Fourier-transform-limited pulses, linearly chirped pulses, and pulses that have experienced self-phase modulation (but no group-velocity dispersion). We take the pulse intensity envelope to be Gaussian for all cases, which reasonably approximates many ultrashort pulse shapes. It is proved that phase variations within the pulse have a strong effect on the correlation function and thus on the IGA trace.

A. Chirped Pulses

Consider first a linearly chirped Gaussian-intensity pulse:

$$E(t) = A_0 \exp[-i\omega_0 t - 2(\ln 2)(t^2/{\tau_p}^2 + i\alpha t^2)], \quad (19)$$

where τ_p is the FWHM of the pulse intensity, α is the chirp parameter, and the instantaneous frequency is given by $\omega = \omega_0 + 4(\ln 2)\alpha t$. By direct integration $k(\tau)$ is found to be

$$k(\tau) = {A_0}^2 \frac{\tau_p}{2}\left(\frac{\pi}{\ln 2}\right)^{1/2} \exp[-(\ln 2)(1 + \alpha^2{\tau_p}^4)(\tau^2/{\tau_p}^2)] \times \exp(-i\omega_0\tau). \quad (20)$$

The IGA trace will be proportional to $|k(\tau)|^2$, which is a Gaussian with a FWHM $\Delta\tau = \sqrt{2}\tau_p/(1 + \alpha^2{\tau_p}^4)^{1/2}$. The intensity spectrum of the pulse can be found by computing the transform of $k(\tau)$. The FWHM of this frequency spectrum, Δf, satisfies the relation $\Delta f \Delta t = 2\sqrt{2}(\ln 2)/\pi = 0.62405$.

Introducing the normalized delay $x = \tau/\tau_p$ and normalizing the correlation function so that $k(0) = 1$, we have

$$|k(x)|^2 = \exp[-2(\ln 2)x^2(1 + \alpha^2{\tau_p}^4)]. \quad (21)$$

Figure 3 shows IGA traces for linearly chirped pulses as the chirp parameter is varied from $\alpha{\tau_p}^2 = 0$ (transform

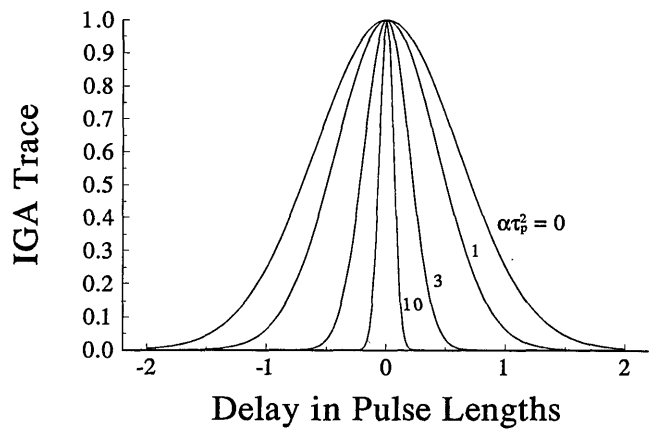


Fig. 3. IGA traces $[|k(x)|^2]$ for linearly chirped pulses. The chirp parameters used were $\alpha{\tau_p}^2 = 0$ (transform limited), $\alpha{\tau_p}^2 = 1$, $\alpha{\tau_p}^2 = 3$, and $\alpha{\tau_p}^2 = 10$. Note that the IGA trace becomes narrower as the chirp parameter increases (for a given pulse width).

limited) to $\alpha\tau_p^2 = 10$ (heavily chirped). The narrowing of the trace is clearly apparent. Since an SHG autocorrelation trace will be identical to one for the $\alpha = 0$ case, the combination of SHG and IGA can provide an accurate measurement of both the pulse length and the chirp parameter $\alpha\tau_p^2$.

B. Self-Phase-Modulated Pulses

We now consider what happens when a Gaussian pulse with no chirp passes through a nondispersive, nonlinear medium of length z. In the simple theory of self-phase modulation (SPM),[57] the intensity of the pulse retains its Gaussian time dependence, but the pulse acquires a time-dependent phase shift of

$$\phi(t) = \omega_0\tau_p Q \exp[-4(\ln 2)(t^2/\tau_p^2)], \tag{22}$$

where Q is the spectral broadening parameter given by

$$Q = \frac{2\pi\omega_0\chi^{(3)}A_0^2 z}{k_0 c^2 \tau_p}. \tag{23}$$

Thus the electric field may be written as

$$E(t) = A_0 \exp[-2(\ln 2)(t^2/\tau_p^2) - i\{\omega_0 t + \omega_0\tau_p Q \exp[-4(\ln 2)(t^2/\tau_p^2)]\}]. \tag{24}$$

Substituting into the expression for $k(\tau)$, we obtain

$$k(x) = 2\left(\frac{\ln 2}{\pi}\right)^{1/2} \int_{-\infty}^{+\infty} du \exp\{-2(\ln 2)u^2 - i\omega_0\tau_p Q \times \exp[-4(\ln 2)u^2]\}\exp\{-2(\ln 2)(u+x)^2 - i\omega_0\tau_p Q \times \exp[-4(\ln 2)(u+x)^2]\}, \tag{25}$$

where we have introduced the normalized time $u = t/\tau_p$ and the normalized delay $x = \tau/\tau_p$ and have normalized the correlation function so that $k(0) = 1$.

We may now examine the significance of the SPM strength, $\omega_0\tau_p Q$, by computing the autocorrelation function $k(x)$ and numerically evaluating its Fourier transform to get the spectrum. We find that, when the SPM strength is above 5, the frequency bandwidth is approximately $\Delta f = 0.45\omega_0 Q$. Thus, with this simple result, the SPM strength $\omega_0\tau_p Q$ together with the pulse width τ_p provides a direct measurement of the spectral broadening produced by the magnitude of the SPM. We show below that the IGA trace alone in fact yields both the SPM magnitude and the pulse length.

Figure 4 shows the computed IGA trace obtained from $|k(x)|^2$ for self-phase-modulated pulses having four different values of the SPM strengths $\omega_0\tau_p Q$. As the SPM strength increases, the central peak narrows in the same manner as for increasing chirp. In addition the wings develop sidelobes that are a clear signature of SPM. A quick estimate of the SPM strength can be obtained by measurement of the location of the first zero (denoted r_1) and by use of the approximation that $\omega_0 r_1 Q \approx 1.9$. A fast way of obtaining the frequency bandwidth is to measure the width of the central peak of the IGA trace (denoted ΔT) and to use the approximation that $\Delta T \Delta f \approx 0.93$. This approximation is particularly good for SPM strengths above 5.

We now show that the IGA trace alone in fact yields both the SPM magnitude and the pulse length. Recall that the IGA trace for a chirped pulse cannot simultaneously yield the pulse length and the chirp parameter. This is because the IGA trace for a chirped Gaussian pulse is always Gaussian, and a narrow trace can result from a short pulse length or a large chirp parameter. This is a fairly unique situation, and it is not the case for IGA traces of self-phase modulated pulses. While it is still the case that a narrow trace can result from a short pulse length or a large bandwidth owing to a large amount of SPM, the IGA trace shapes in the two cases are quite different. A narrow trace with no oscillations in the wings indicates a short pulse, and a narrow trace with many oscillations in the wings indicates a long pulse that has experienced much SPM. Thus it is simply necessary to fit

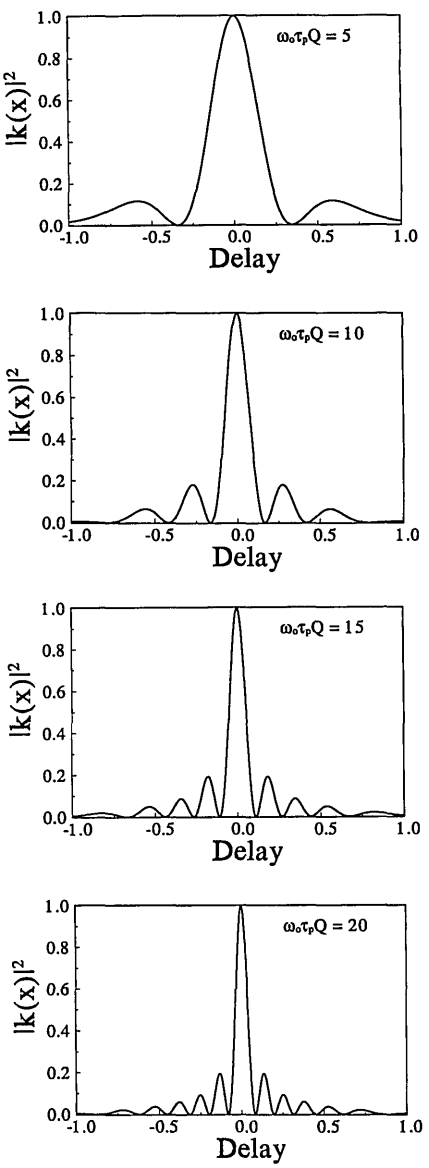


Fig. 4. Theoretical IGA traces $|k(x)|^2$ for SPM pulses. The SPM strengths $\omega_0\tau_p Q$ shown are 5, 10, 15, and 20. Note that the IGA trace becomes narrower and develops oscillations in the wings as the amount of SPM increases.

the trace to Eq. (25) for exact results or to estimate these parameters from the rules of thumb given above and below for rough results. Of course, for small amounts of SPM, no oscillations will occur in the IGA trace, yielding a threshold for the utility of the method (that value of SPM at which the spectrum starts to split; see below). On the other hand, if the SPM can be varied, the relative width of the trace can be used to measure the SPM much more sensitively.

Because the IGA trace is equal to $|k(x)|^2$, there is value in studying the mathematical properties of the function $k(x)$. For a symmetric pulse experiencing SPM and whose intensity is an even function of time, the correlation function $k(x)$ will be even and real. Furthermore, the value of $k(x)$ for large x approaches zero from the positive side. For small values of the SPM strength $\omega_0\tau_p Q$, $k(x)$ is positive definite for all x. At the SPM strength of 2.5705, $k(x)$ becomes tangent to the x axis. Thus increasing $\omega_0\tau_p Q$ slightly above this value introduces two zeros into the function. As the SPM strength is increased further, additional pairs of zeros are introduced. Some critical values of $\omega_0\tau_p Q$ at which a new pair of zeroes is added are 8.7956, 15.0375, 21.2967, 27.5647, and 33.8463. For large values of $\omega_0\tau_p Q$, increasing $\omega_0\tau_p Q$ by 2π rad introduces an extra pair of zeros.

We can now understand the reason for the oscillatory wings. The self-phase modulation introduces a time-dependent phase shift whose magnitude depends on $\omega_0\tau_p Q$. Delaying one signal by an appropriate amount can force the relative phase shifts either to add or to cancel. The number of specific delays for which exact cancellation is observed depends on the magnitude of $\omega_0\tau_p Q$. There is no simple expression for the location of these zeros, and they can be obtained only by numerical evaluation of the complicated integral given in Eq. (25).

C. Other Phase Distortions

Just as ISHGA is not a general diagnostic for phase, IGA does not reveal all phase distortions present in a pulse. Recall that the IGA trace is the magnitude-squared Fourier transform of the pulse spectrum. Thus pure frequency-domain phase distortions, such as the well-known frequency-domain cubic phase $[\varphi(\omega) = a\omega^3]$, in principle leave the pulse spectrum unchanged and so will be undetectable from only the IGA trace. Additional information (such as an autocorrelation or a frequency-resolved-optical-grating trace) is then necessary to indicate such distortions.

For the same reason phase distortions that wash out any structure in the spectrum will wash out structure in the IGA trace. For example, the IGA trace of a SPM pulse has oscillatory structure in its wings, which is due to the oscillatory structure in the pulse spectrum. If some other phase distortion causes this spectral structure to wash out, then the oscillatory wings of the IGA trace for this pulse will also wash out. Thus, when SPM is also accompanied by group-velocity dispersion, the resulting pulse is essentially linearly chirped, and we have seen that such a pulse has no oscillatory structure in its IGA trace.[33] The same is also true for the envelope of the high-frequency fringes of the ISHGA trace, which is related to the IGA trace.

5. COMPARISON WITH EXPERIMENTS

The experimental apparatus is shown in Fig. 5. A cw harmonically mode-locked Nd:YAG laser generated a 100-MHz train of nominal 50-ps duration pulses at 1.064 μm. In the IGA experiments the optical pulses were split in a modified Michelson interferometer, with one arm of fixed optical delay and the other delayed by a stepper-motor–retroreflector combination. The two beams were focused into the photorefractive medium by a pair of 15-cm focal-length lenses at an intersection angle of $2\theta = 40°$, which resulted in a grating period of 1.5 μm. The resultant beam diameter in the photorefractive medium was approximately 100 μm. One beam, designated the pump beam, was chopped, and the transfer of modulation was detected in the probe beam with a silicon p-i-n photodiode and a lock-in amplifier. This is a standard photorefractive beam-coupling geometry except for the variable optical delay at one of the interferometer arms. The IGA response was determined by measurement of the lock-in amplifier output as a function of the delay between the pump and the probe beams.

Our goal was to check the detailed prediction of the theory using self-phase-modulated pulses. To produce well-defined self-phase-modulated pulses, the light was propagated through a 24-m length of polarization-maintaining silica fiber with an effective core area of 5×10^{-7} cm^2. For this length of fiber and the 1.06-μm wavelength, group-velocity dispersion was negligible. The SPM strength $\omega_0\tau_p Q$ was varied by variation of the average power of the pulses.

In these experiments we used classic photorefractive media, the ferroelectric oxides $KNbO_3$ and $BaTiO_3$. At 1.06 μm these crystals provided a unique measurement capability. Since this wavelength is in the transparency region of the crystal, a standard SHGA pulsewidth measurement may be done at the same time as the IGA measurement. By placing the silicon photodetector at the bisector of the pump and the probe beams, we were able to perform a noncollinear background-free SHGA measurement. The crystal orientation was optimized for maximum photorefractive beam coupling and was therefore not phase matched for SHG. Despite this fact, the input beams were intense enough to generate a visible (green, 532 nm) SHG signal and to yield an intensity autocorrelation easily. Thus with a single crystal one can simultaneously measure IGA and SHGA traces and obtain information on both the bandwidth and the pulse width of an ultrashort optical pulse in the time domain.

A typical experimental result of using SHGA to measure the pulse width is shown in Fig. 6. The smooth curve is a Gaussian curve that fits the data shown by

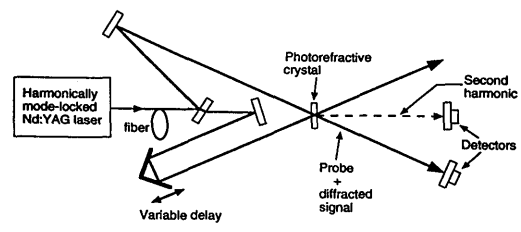


Fig. 5. Experimental apparatus, a standard photorefractive-beam-coupling arrangement, but with SHG possible simultaneously.

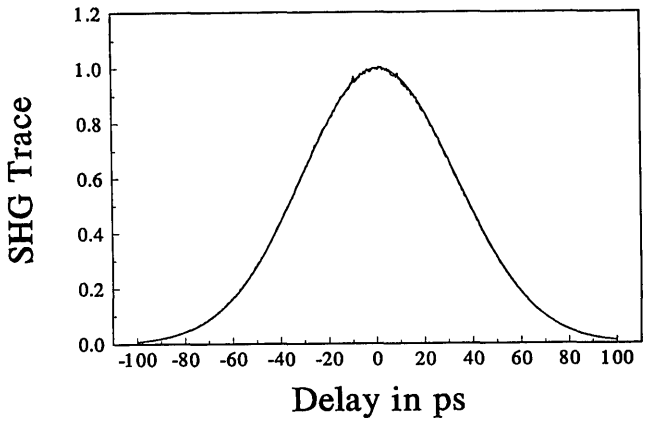


Fig. 6. Simultaneous SHGA measurement made with the same apparatus and crystal as in photorefractive-beam-coupling IGA measurements.

the noisy curve. The fit is extraordinarily good, even in the far wings of the pulse. This justifies the assumption made in the theoretical derivation that the pulses have a Gaussian-intensity envelope. From this fit the pulse length was determined to be 53 ps.

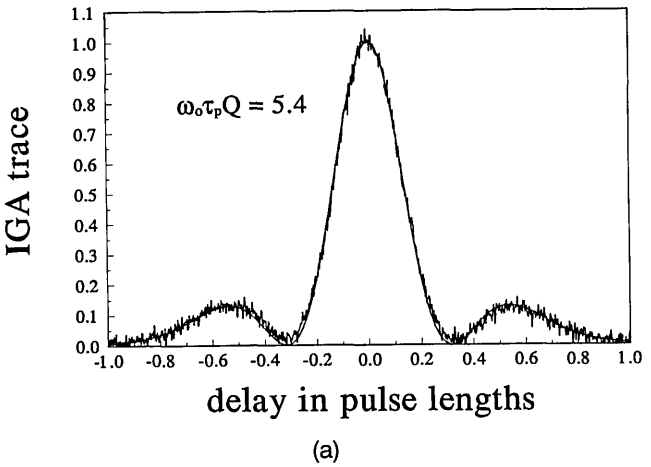
Some typical IGA traces are shown in Fig. 7. The traces shown are for powers of 0.25, 0.5, and 1.0 W. Also shown in the same figure are smooth curves computed from Eq. (25) for the values of $\omega_0\tau_pQ$ indicated. We note that the measured values of $\omega_0\tau_pQ$ were proportional to the power, as expected. The agreement observed in the three traces shown was typical for all the data taken in these experiments. Although these traces were obtained with $KNbO_3$, similar results were obtained with $BaTiO_3$. The excellent agreement in these traces indicates the utility of IGA for pulse measurement, in agreement with independent SHGA measurements.

We also performed detailed fits to these traces to determine the pulse length, τ_p. In all experiments performed to date, the value of τ_p obtained was within 15% of the value measured by SHGA. This demonstrates that it is indeed possible to measure the SPM magnitude and the pulse length from the IGA trace.

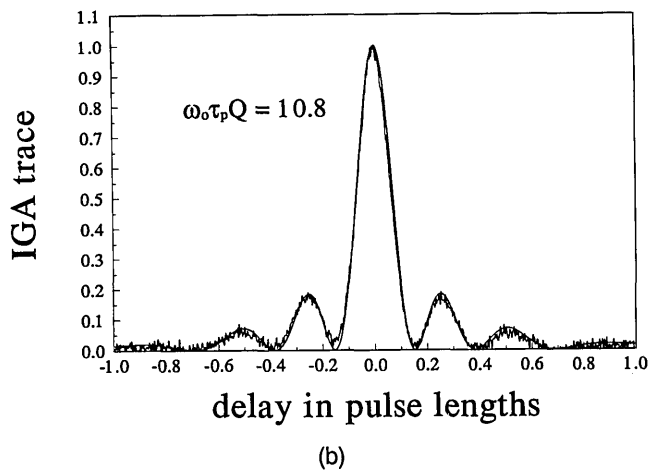
6. DISCUSSION AND CONCLUSIONS

In this paper we have derived expressions for the IGA trace observed in two- and three-beam geometries for single-pulse experiments and in all geometries in many-pulse accumulated-grating experiments. In all these cases the IGA trace is the squared magnitude of the electric-field autocorrelation function. The remaining case (single-pulse self-diffraction) yields a more complicated expression that is not suitable for our purposes. Since the electric-field autocorrelation function is the Fourier transform of the spectrum, simultaneous measurement of the intensity autocorrelation function (for example, by SHGA) and the field autocorrelation function (by IGA) provides an easy approach for determining both pulse duration and pulse bandwidth entirely in the time domain.

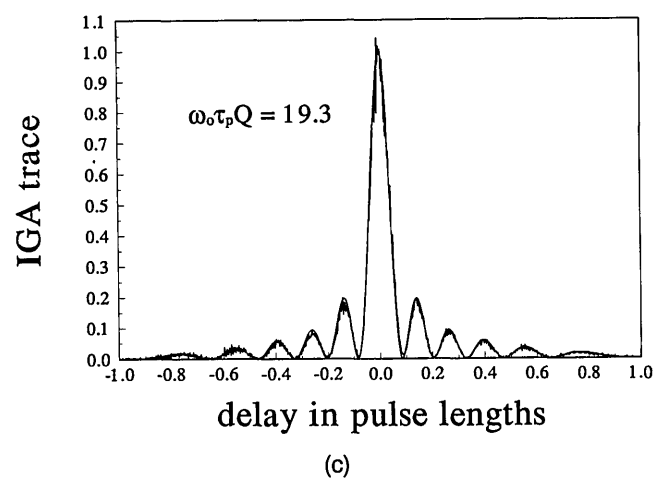
To validate the theoretical expressions, we have considered in detail the case of self-phase-modulated pulses. As is shown above, the electric-field autocorrelation function is complicated, involving multiple zeros symmetrically placed around a central peak. The final expression for the IGA trace can be evaluated only by numerical integration. This theoretical expression was derived with the assumption that the intensity of the pulse has a Gaussian time dependence.



(a)



(b)



(c)

Fig. 7. Typical experimental IGA traces for powers of (a) 0.25 W, (b) 0.5 W, and (c) 1.0 W. As power into the fiber increases, the magnitude of the SPM increases, and the IGA trace narrows and contains more oscillations in the wings. Also shown is the fit to the data from Eq. (25).

As predicted by the theory, the value of $\omega_0\tau_p Q$ measured by IGA was proportional to the average power. More impressive was the detailed agreement between theory and experiment on both the location of the zeros and the relative heights of the sidelobes in the IGA traces. We believe that this was extraordinary, given that the only fitting parameters were the height and the center of the main peak, the pulse length, and the SPM parameter $\omega_0\tau_p Q$.

We also demonstrated that the IGA measurement can be used to obtain both the pulse width and the spectral broadening that is due to SPM in a single measurement. Although this is not possible in general (linear chirp is an important counterexample), if a model is available for the pulse some pulse-length information is generally available in the IGA trace.

In this paper we have been interested in confirming that the theoretical expressions for the IGA traces are correct, and we have verified this to be the case. We used an optical fiber to generate well-defined self-phase-modulated pulses. In the future, however, we will explore the use of the IGA measurements to characterize the nonlinear properties of new fibers.

ACKNOWLEDGMENTS

This research was partially supported by the U.S. Department of Energy, Office of Basic Energy Sciences, Chemical Sciences Division. We thank Marvin Klein (Hughes Research Laboratories) for the loan of the photorefractive crystals, Roger Stolen and Ken DeLong for helpful discussions, and G. Michalakeas and H. Zerouali for their assistance.

REFERENCES

1. E. P. Ippen and C. V. Shank, in *Ultrashort Light Pulses—Picosecond Techniques and Applications*, S. L. Shapiro, ed. (Springer-Verlag, Berlin, 1977), pp. 83–122.
2. J. A. Giordmaine, P. M. Rentzepis, S. L. Shapieo, and K. W. Wecht, "Two-photon excitation of fluorescence by picosecond light pulses," Appl. Phys. Lett. **11,** 216–218 (1967).
3. J. I. Dadap, G. B. Focht, D. H. Reitze, and M. C. Downer, "Two-photon absorption in diamond and its application to ultraviolet femtosecond pulse-width measurement," Opt. Lett. **16,** 499–501 (1991).
4. N. G. Basov, V. E. Pozhar, and V. I. Pustovoit, "Measurement of the duration of high-power ultrashort optical pulses," Sov. J. Quantum Electron. **15,** 1429–1431 (1985).
5. J. P. Bernardin and N. M. Lawandy, "Picosecond pulse measurements using the active laser medium," IEEE J. Quantum Electron. **26,** 399–402 (1990).
6. O. L. Bourne and A. J. Alcock, "Ultraviolet and visible single-shot autocorrelator based on multiphoton ionization," Rev. Sci. Instrum. **57,** 2979–2982 (1986).
7. J. C. Diels, J. J. Fontaine, and F. Simoni, "Phase sensitive measurements of femtosecond laser pulses from a ring cavity," in *Proceedings of the International Conference on Lasers* (STS, McLean, Va., 1983), pp. 348–355.
8. J. C. M. Diels, J. J. Fontaine, I. C. McMichael, and F. Simoni, "Control and measurement of ultrashort pulse shapes (in amplitude and phase) with femtosecond accuracy," Appl. Opt. **24,** 1270–1282 (1985).
9. J. C. Diels and J. J. Fontaine, "Coherence properties of ultrashort optical pulses," J. Opt. (Paris) **16,** 115–119 (1985).
10. J. C. Diels, "Measurement techniques with mode-locked dye laser," in *Ultrashort Pulse Spectroscopy and Applications*, M. J. Soileau, ed., Proc. Soc. Photo-Opt. Instrum. Eng. **533,** 63–70 (1985).
11. C. Yan and J. C. Diels, "Amplitude and phase recording of ultrashort pulses," J. Opt. Soc. Am. B **8,** 1259–1263 (1991).
12. K. Naganuma, K. Mogi, and H. Yamada, "Time direction determination of asymmetric ultrashort optical pulses from second-harmonic generation autocorrelation signals," Appl. Phys. Lett. **54,** 1201–1202 (1989).
13. K. Naganuma, K. Mogi, and H. Yamada, "General method for ultrashort light pulse chirp measurement," IEEE J. Quantum Electron. **25,** 1225–1233 (1989).
14. E. B. Treacy, "Measurement and interpretation of dynamic spectrograms of picosecond light pulses," J. Appl. Phys. **42,** 3848–3858 (1971).
15. M. Vampouille, A. Barthélémy, B. Colombeau, and C. Froely, "Observation et applications des modulations de fréquence dans les fibres unimodales," J. Opt. (Paris) **15,** 385–390 (1984).
16. J. E. Rothenberg and D. Grischkowski, "Subpicosecond transient excitation of atomic vapor and the measurement of optical phase," J. Opt. Soc. Am. B **3,** 1235–1238 (1986).
17. J. E. Rothenberg and D. Grischkowski, "Measurement of optical phase with subpicosecond resolution by time-domain interferometry," Opt. Lett. **12,** 99–101 (1987).
18. F. Reynaud, F. Salin, and A. Barthelemy, "Measurement of phase shifts introduced by nonlinear optical phenomena on subpicosecond pulses," Opt. Lett. **14,** 275–277 (1989).
19. T. F. Albrecht, K. Seibert, and H. Kurz, "Chirp measurement of large-bandwidth femtosecond optical pulses using two-photon absorption," Opt. Commun. **84,** 223–227 (1991).
20. A. S. L. Gomes, V. L. da Silva, and J. R. Taylor, "Direct measurement of nonlinear frequency chirp of Raman radiation in single-mode optical fibers using a spectral window method," J. Opt. Soc. Am. B **5,** 373–379 (1988).
21. F. Salin, P. Georges, and A. Brun, "Complex pulse evolution in a femtosecond laser with spectral windowing," Opt. Commun. **79,** 443–447 (1990).
22. K. W. DeLong and J. Yumoto, "Chirped light and its characterization using the cross-correlation technique," J. Opt. Soc. Am. B **9,** 1593–1604 (1992).
23. J. L. A. Chilla and O. E. Martinez, "Direct determination of the amplitude and the phase of femtosecond light pulses," Opt. Lett. **16,** 39–41 (1991).
24. J. L. A. Chilla and O. E. Martinez, "Analysis of a method of phase measurement of ultrashort pulses in the frequency domain," IEEE J. Quantum Electron. **27,** 1228–1235 (1991).
25. J. L. A. Chilla and O. E. Martinez, "Frequency domain phase measurement of ultrashort light pulses. Effects of noise," Opt. Commun. **89,** 434–440 (1992).
26. D. J. Kane and R. Trebino, "Single-shot measurement of the intensity and phase of an arbitrary ultrashort pulse by using frequency-resolved optical gating," Opt. Lett. **18,** 823–825 (1993).
27. D. J. Kane and R. Trebino, "Characterization of arbitrary femtosecond pulses using frequency-optical gating," IEEE J. Quantum Electron. **29,** 571–579 (1993).
28. R. Trebino and D. J. Kane, "Using phase retrieval to measure the intensity and phase of ultrashort pulses: frequency-resolved optical gating," J. Opt. Soc. Am. A **10,** 1101–1111 (1993).
29. K. W. DeLong and R. Trebino, "Improved ultrashort-pulse retrieval algorithm for frequency-resolved optical gating," J. Opt. Soc. Am. A **11,** 2429–2437(1994).
30. R. Trebino, E. K. Gustafson, and A. E. Siegman, "Fourth-order partial-coherence effects in the formation of integrated-intensity gratings with pulsed light sources," J. Opt. Soc. Am. B **3,** 1295–1304 (1986).
31. B. S. Wherrett, A. L. Smirl, and T. F. Boggess, "Theory of degenerate four-wave mixing in picosecond excitation-probe experiments," IEEE J. Quantum Electron. **QE-19,** 680–690 (1983).
32. V. Dominic, X. S. Yao, R. M. Pierce, and J. Feinberg, "Measuring the coherence length of mode-locked laser pulses in real time," Appl. Phys. Lett. **56,** 521–523 (1983).
33. R. Trebino, C. C. Hayden, A. M. Johnson, W. M. Simpson, and A. M. Levine, "Chirp and self-phase modulation in induced-grating autocorrelation measurements of ultrashort pulses," Opt. Lett. **15,** 1079–1081 (1990).
34. H. J. Eichler, U. Klein, and D. Langhans, "Coherence time

measurement of picosecond pulses by a light-induced-grating method," Appl. Phys. **21,** 215–291 (1980).
35. W. L. J. Nighan, T. Gong, L. Liou, and P. M. Fauchet, "Self-diffraction: a new method for characterization of ultrashort laser pulses," Opt. Commun. **69,** 339–344 (1989).
36. X. Zhu, K. Spears, and J. Serafin, "Ultrashort pulsed laser coherence measurements by single-pulse holography and four-wave mixing," J. Opt. Soc. Am. B **6,** 1356–1362 (1989).
37. H. Schulz, H. Schuler, T. Engers, and D. von der Linde, "Measurement of intense ultraviolet subpicosecond pulses using degenerate four-wave mixing," IEEE J. Quantum Electron. **25,** 2580–2585 (1989).
38. J. Etchepare, G. Grillon, and A. Orszag, "Third order autocorrelation study of amplified subpicosecond laser pulses," IEEE J. Quantum Electron. **19,** 775–778 (1983).
39. W. Leupacher and A. Penzkofer, "Temporal analysis of a mode-locked Nd-glass laser by four-wave mixing (in a medium with instantaneous response)," Appl. Phys. B **29,** 263–267 (1982).
40. H. S. Albrecht, P. Heist, J. Kleinschmidt, D. van Lap, and T. Schroder, "Measurement of ultraviolet femtosecond pulses using the optical Kerr effect," Appl. Phys. B **55,** 362–364 (1992).
41. D. M. Rayner, P. A. Hackett, and C. Willis, "Ultraviolet laser, short pulse-width measurement by multiphoton ionization autocorrelation," Rev. Sci. Instrum. **53,** 537–538 (1982).
42. N. Sarukura, M. Watanabe, A. Endoh, and S. Watanabe, "Single-shot measurement of subpicosecond KrF pulse width by three-photon fluorescence of the XeF visible transition," Opt. Lett. **13,** 996–998 (1988).
43. N. Morita and T. Yajima, "A nonlinear correlation method using multiphoton ionization for the measurement of UV ultrashort pulses," Appl. Phys. B **28,** 25–29 (1982).
44. J. Janszky, G. Corradi, and D. S. Hamilton, "Temporal analysis of short laser pulses using degenerate four-wave mixing," Appl. Opt. **23,** 8–9 (1984).
45. J. Janszky and G. Corradi, "Full intensity profile analysis of ultrashort laser pulses using four-wave mixing or third harmonic generation," Opt. Commun. **60,** 251–256 (1986).
46. A. M. Johnson, A. M. Glass, W. M. Simpson, B. Bylsma, and D. H. Olson, "Microwatt picosecond pulse autocorrelator using photorefractive GaAs:Cr," in *Annual Meeting*, Vol. 11 of 1988 OSA Technical Digest Series (Optical Society of America, Washington, D.C., 1988), paper ThC4.
47. A. M. Johnson, A. M. Glass, W. M. Simpson, and D. H. Olson, "Infrared picosecond pulse diagnostics using photorefractive beam coupling," in *Conference on Lasers and Electro-Optics*, Vol. 11 of 1989 OSA Technical Digest Series (Optical Society of America, Washington, D.C., 1989), p. 226.
48. A. M. Johnson, W. M. Simpson, A. M. Glass, M. B. Klein, D. Rytz, and R. Trebino, "Infrared picosecond pulse correlation measurements using photorefractive beam coupling and harmonic generation in $KNbO_3$ and $BaTiO_3$," in *Annual Meeting*, Vol. 18 of 1989 OSA Technical Digest Series (Optical Society of America, Washington, D.C., 1989), p. 53.
49. X. S. Yao and J. Feinberg, "Photorefractive pulse coupling in the frequency domain," Opt. Lett. **18,** pp. 104–106 (1993).
50. R. Baltrameyunas, Y. Zaitkus, R. Dannelyus, M. Pyatrauskas, and A. Piskarskus, "Applications of dynamic holography in determination of coherence times of single picosecond light pulses," Sov. J. Quantum Electron. **12,** 1252–1254 (1982).
51. J. Buchert, R. Dorsinville, P. Delfyett, S. Krimchansky, and R. R. Alfano, "Determination of thermal correlation of ultrafast laser pulses using phase conjugation," Opt. Commun. **52,** 433–437 (1985).
52. P. M. Fauchet, W. L. Nighan, Jr., and R. Trebino, "Characterization of ultrashort laser pulses by the method of self-diffraction," in *Advances in Laser Science—I, Proceedings of the First International Laser Science Conference*, W. C. Stwalley and M. Lapp, eds. (American Institute of Physics, New York, 1986), pp. 588.
53. A. M. Levine, E. Ozizmir, R. Trebino, and C. C. Hayden, "New developments in autocorrelation measurements of ultrashort pulses," in *Laser Spectroscopy X*, M. Ducloy, E. Giacobino, and G. Camy, eds. (World Scientific, Singapore, 1992), pp. 384–385.
54. G. Szabo, Z. Bor, and A. Muller, "Phase-sensitive single-pulse autocorrelator for ultrashort laser pulses," Opt. Lett. **13,** 746–748 (1988).
55. S. P. Le Blanc, G. Szabo, and R. Sauerbrey, "Femtosecond single-shot phase-sensitive autocorrelator for the ultraviolet," Opt. Lett. **16,** 1508–1510 (1991).
56. X. S. Yao, V. Dominic, and J. Fainberg, "Theory of beam coupling and pulse shaping of mode-locked laser pulses in a photorefractive crystal," J. Opt. Soc. Am. B **7,** 2347–2355 (1990).
57. Y. R. Shen and G.-Z. Yang, "Theory of self-phase modulation and spectral broadening," in *The Supercontinuum Laser Source*, R. R. Alfano, ed. (Springer-Verlag, Berlin, 1989), pp. 1–32.