Standards for ultrashort-laser-pulse-measurement techniques and their consideration for self-referenced spectral interferometry

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Issues important for new ultrashort-pulse-measurement techniques include the generation of theoretical example traces for common pulses, validity ranges, ambiguities, coherent artifacts, device calibration sensitivity, iterative retrieval convergence, and feedback regarding measurement accuracy. Unfortunately, in the past, such issues have gone unconsidered, yielding long histories of unsatisfactory measurements. We review these issues here in the hope that future proposers of new techniques will consider them without delay, and, as an example, we address them for a relatively new technique: self-referenced spectral interferometry. © 2014 Optical Society of America

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1. Introduction

The ability to accurately measure ultrashort laser pulses is essential to creating, using, and improving them, but the technology for their measurement has consistently lagged behind that for their generation. The result has been a long and sometimes quite painful history of attempts—and failures—to measure these exotic and ephemeral events. The reason is that many pulse-measurement techniques have suffered from, and continue to suffer from, a wide range of complications, including the presence of ambiguities, insufficient temporal and/or spectral resolution and/or range, an inherent inability to measure the complete pulse intensity and/or phase, an inability to measure complex pulses, and misleading results due to the loss of information due to idiosyncrasies of the technique or multishot averages over different pulses.

The most famous complication occurred almost immediately after the birth of the field of pulse measurement in the 1960s and involved the first pulse-measurement technique, intensity autocorrelation [1–3]. It was obvious from its inception that intensity autocorrelation would not determine the phase of pulses. And it was also known that, in addition to some trivial intensity ambiguities, such as time-reversal, intensity autocorrelation also had many *nontrivial* intensity ambiguities with, among other differences, very different pulse lengths. This is because the autocorrelation theorem implies that retrieving the actual pulse intensity versus time from its intensity autocorrelation is mathematically equivalent to the well-known, highly ill-posed one-dimensional phase-retrieval problem. Worse, the nontrivial ambiguities for a given autocorrelation—usually infinite in number—generally cannot be determined [4]. Worse still, intensity autocorrelation also has many approximate ambiguities whose autocorrelation traces are too similar to be distinguished experimentally [5]. Furthermore, as the

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pulse intensity increases in complexity and structure, the intensity autocorrelation actually becomes simpler and less structured (see Fig. 1), clearly losing most of the information about a potentially complex pulse and approaching a very simple shape in the limit of infinite complexity [6]. This shape consists of a narrow central spike atop a broad background. As it is related to the coherence length in linear correlation measurements [7], the narrow central spike has come to be known as the *coherence spike*, or *coherent artifact*, and it indicates only the shortest temporal structure in the potentially much more complex pulse intensity. For extremely complicated pulses, the width of this spike is the coherence time of the pulse [8,9]. To complicate matters further, the coherent artifact is not limited to single-shot autocorrelations of complex pulses, but also appears in multishot averages over *unstable* trains of pulses. Averaging over many different pulses tends to emphasize the coherence spike in the autocorrelation trace, often yielding a trace with a small background. This background is often ignored, even though it is actually the correct indicator of the overall average pulse length. As in single-shot autocorrelations, the multishot coherence spike also represents the smallest repeating (i.e., coherent) substructure common to all pulses.

The coherent artifact has been especially problematic in several prototypical situations. One of these is mode locking of lasers with short upper-state lifetime, as is typical for dye lasers [10,11]. While mode-locked dye lasers play only a marginal role today, mode-locked semiconductor lasers are playing an ever-increasing role in laboratories around the world and display similarly vivid dynamics [12,13]. A second typical scenario is pulse compression in the anomalous dispersion regime [14–16]. Launching high-order solitons into fibers or anomalous bulk media inevitably leads to a rapid decay into fundamental solitons, a process that has been termed soliton fission in the fiber community [17,18]. A third example is the tendency toward multipulsing in some ultrafast fiber lasers, which must be avoided carefully to achieve maximum pulse energy [19,20]. It should also be noted that dynamical instabilities have also been observed in systems that are often considered completely benign and predictable by most researchers. One such example is Kerr-lens mode locking, which shows the occasional appearance of subharmonics [21,22]. Not only is pulse-shape instability an inevitable factor in some areas of current research interest, but it also has the potential to appear in many more scenarios than first anticipated. As a result, the ability to recognize a coherent artifact in situations where it is not expected is important. The burden of proof that a laser is stable is on the measurer.

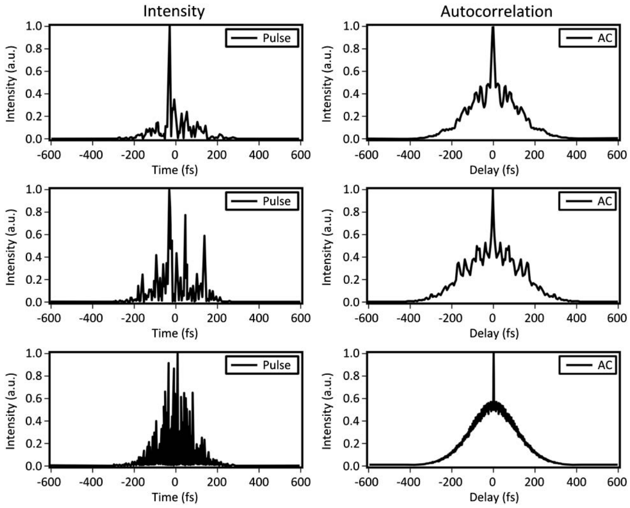

Fig. 1. Single-shot intensity autocorrelations of pulses of increasing complexity [5]. Very complex pulses actually have very simple autocorrelations. Note the coherence spike in each autocorrelation, which indicates only the coherence time of the pulse, and not the pulse length. This coherence time indicates the characteristic temporal modulation period within the much wider enveloping structure of the pulse.

Despite the availability of simpler, more reliable, and more informative techniques, autocorrelation remains popular even today. And while it is straightforward to identify a coherence spike if its width is an order or magnitude or more narrower than the enveloping pulsewidth, situations exist in which the coherent artifact is far more difficult to detect. The problem is compounded by the fact that many commercial autocorrelators available today provide users the ability to manually subtract off arbitrary amounts of background, encouraging them to leave behind *only* the coherent artifact and potentially a significant underestimate of the pulse length. As a result, even an autocorrelation measurement of a continuous-wave laser may be misinterpreted as a short pulse. Although the multishot coherent artifact in autocorrelation was first identified back in 1969 [23], it remains surprisingly problematic even today. Even at this moment, debate rages over whether a recent autocorrelation measurement is simply a coherent artifact or not [13,24,25], leading one to legitimately question how much progress has actually been made in the field of pulse measurement since the mid-1960s.

One reason for the continued presence of coherent artifact issues is that autocorrelation (and many other later methods) gives little or no feedback that a measurement has retrieved a correct or incorrect result, so users may not be aware that their measurement is incorrect. This is not uncommon in optics, as, for example, spectrometers and power meters also offer no such feedback. All that intensity autocorrelation can offer in this regard is symmetry with respect to delay and a peak at zero delay. Autocorrelation's interferometric cousin [26] additionally offers an 8-1 peak-to-background ratio. Unfortunately, these checks are not very strong, and even when they are satisfied, most of the previously mentioned misinterpretations remain possible. Unless a technique gives some clear feedback when it fails to correctly measure a pulse, a user of such a technique is almost certain to be led to believe that he has generated very short, simple, and stable (typically desirable) pulses when he has in fact generated long, complex, and unstable (typically undesirable) ones. Some, but not all, modern pulse-measurement techniques offer feedback. Ideally, presenting and analyzing measurement feedback would always be a priority.

While researchers have come to expect that a new measurement technique should, in fact, determine the intensity and phase of pulses, rarely is a proposed new technique checked for the coherent artifact. Furthermore, few, if any, of the other issues mentioned above are promptly investigated for new measurement techniques. For an initial publication, only some demonstration of the technique's ability to measure a known phase is typically required, leaving an investigation of ambiguities, calibration issues, coherent-artifact effects, and other easy-to-check problems for future work that may never occur. Indeed, this has been the case for many pulse-measurement techniques in use today. For example, while many of the above potential complications were studied fairly early for frequency-resolved optical gating (FROG) [27,28], its coherent artifact was not considered until almost a decade after its introduction, when it was observed in multishot continuum measurements [29]. It was found that FROG performed well in its presence, with the algorithm ignoring the coherent artifact and retrieving a representative (extremely complex) pulse, and discrepancies between the measured and retrieved traces providing a clear indication of its presence and associated instability. As FROG is a correlation-based method, it reacts to a coherence spike in a manner similar to autocorrelation measurements, with a background that contains useful information about the pulse. Methods based on spectral interferometry, in contrast, have not dealt with this issue as well, yielding less useful backgrounds. The SPIDER method [30–32], for example, which has been used for thousands of pulse measurements over more than 15 years, was shown only recently to measure *only* the coherent artifact, with very little feedback that anything is wrong with the measurement [33,34]. SPIDER certainly delivers accurate results when the laser is known to be stable, and most commercial ultrafast lasers are usually stable when properly aligned. But what was only recently realized is that a SPIDER measurement typically says nothing about *whether* the laser is stable—something that is essentially impossible to establish without resorting to an alternative pulse-measurement technique. In other words, SPIDER cannot distinguish a stable train of short pulses from an unstable train of much longer pulses. So it can erroneously indicate a pulse significantly shorter, simpler, and more stable than is actually the case. Effects like this that may have a significant impact on interpretation of measurements should be identified as quickly as possible.

It is often considered a safeguard to characterize pulses with several methods simultaneously [32,35]. However, if the employed techniques all have the same issue, such as susceptibility to the coherent artifact (e.g., because they are both based on spectral interferometry), even perfect agreement among them of the retrieved pulse shape does not shield against problems such as the coherent artifact. Unfortunately, little has been published about this particular susceptibility for most measurement techniques. Worse, little is known about the other above-mentioned complications that routinely plague such measurements. Because progress in the generation and application of ever shorter, simpler, and more stable pulses depends sensitively on the availability of reliable and robust measurement techniques, it is critical to develop and, more importantly, to *understand* new techniques and their possible inadequacies.

It is clear—especially in view of the difficult history of ultrashort-pulse measurement—that a fairly thorough analysis of a proposed new technique should be performed *before* it enters common use. Fortunately, some very simple checks can be performed when a technique is introduced, which can prevent most of the problems that have occurred over the years. So, in this publication, we will describe several simple issues that we consider most important. With the benefit of hindsight, we therefore propose they be considered for any newly proposed ultrashort-pulse-measurement technique and also for any existing technique for which they have not yet been considered. As an example, we will apply them to a relatively new measurement technique: self-referenced spectral interferometry (SRSI) using cross-polarized wave generation (XPW) [36], which, to its inventors' credit, has been reasonably well characterized compared to other methods. To accomplish this, we have summarized known results, but, where they are missing, we have performed new simulations of this method's performance and include them to demonstrate the proposed standards. The next section will provide a quick overview of the basics of this pulse-measurement technique, and subsequent sections will outline the standards and recommendations that we propose.

2. Background on SRSI

SRSI is an extension of a technique that has been known for many years: spectral interferometry. Spectral interferometry measures the spectral phase of an unknown pulse by measuring the spectral fringes created between that pulse and a pulse with a known phase and a relative delay [37]. One of the major limitations of spectral interferometry for ultrashort-pulse measurement is that a reference pulse with an equally wide or broader spectrum and a known phase is required to measure an unknown pulse. For very short pulses, an appropriate reference pulse is often not readily available. SRSI attempts to overcome this limitation by using a nonlinear process known as XPW to create a reference pulse from the unknown pulse. As the name suggests, the pulse generated via this nonlinear interaction has a polarization orthogonal to the polarization of the input pulse, and therefore is easily separated from the input. This effect is third order and automatically phase matched. A good approximation of the XPW reference pulse is

$$\begin{aligned} E_{\mathrm{XPW}}(t) &= |E(t)|^2 E(t) \\ &= \mathcal{FT}^{-1}\{E(\omega) * E^*(-\omega) * E(\omega)\}, \end{aligned} \tag{1}$$

which takes into account all the frequency combinations that contribute to the signal at a given frequency [38]. This process will tend to make a reference pulse that is shorter (or at least has sharper features) in the time domain. In many cases, this reference pulse will also have a broader spectrum with smaller, although still nonzero, phase variations compared to the input pulse. Equation (1) neglects other third-order effects that could potentially occur, such as self-phase modulation and cross-phase modulation. In the limit that the conversion efficiency for XPW generation is very low, this is a reasonable assumption. As in standard spectral interferometry, the input pulse and the reference pulse experience a relative delay, and the resulting spectral fringes are measured by a spectrometer.

Using standard Fourier-transform spectral interferometry (FTSI) techniques [37,39], two spectra and the phase difference can be computed from the spectral interferometry signal. The phase difference measured in the trace is not necessarily equal to the phase of the input because the reference pulse will have some residual phase; however, the true input phase can be retrieved iteratively [40,41] (see Fig. 2). The first step is to estimate the XPW phase by simulating it, using the measured phase difference as the phase of the input pulse. With this estimate for the XPW phase, the input phase can be more accurately taken to be the sum of the measured phase difference and the XPW phase. This more accurate version of the input phase can be used to calculate a better estimate for the XPW phase, and the process iteratively continues until convergence is reached. Since a pulse is completely defined by its intensity and phase in the spectral domain, the pulse has been retrieved once its phase is found. A small issue with this retrieval algorithm is that the simulated pulses sometimes begin to accumulate linear spectral phase. This is very easily corrected by shifting the pulse back to its original temporal location in time on each pass through the algorithm, just before the XPW pulse is calculated.

As this technique is now several years old and has begun to be used to make measurements of significance, we feel that examining its performance in certain key areas of historical difficulty is important and perhaps overdue. Indeed, it provides an excellent example for our proposed standards.

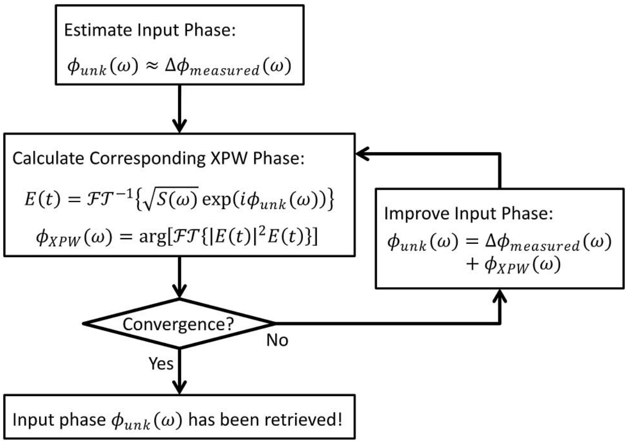

Fig. 2. Phase-retrieval algorithm for SRSI [41]. $S(\omega)$ is the spectrum of the input pulse to be measured, and $\phi_{\mathrm{unk}}(\omega)$ is its phase.

3. The Standards

A. Theoretical Example Measurements

The first task for anyone proposing a new measurement technique should be to generate theoretical example traces for a number of common pulse distortions so that the potential user will know what to expect for measured traces. A good list of effects to include is as follows: positive and negative chirp, cubic spectral phase, quartic spectral phase, and self-phase modulation. In addition, a double pulse and a complex pulse (in which all the spectral components of the pulse have random phases—such as a burst of thermal noise) should be included. Showing these example measurements helps verify that the technique's theoretical basis is sound and promotes understanding. While this sounds obvious, perhaps too obvious even to mention here, such simple plots are in fact *not* routinely computed, and fundamentally impossible techniques have found their way into the archival scientific literature as a result [42,43].

Generating these plots for SRSI shows the expected features (see Fig. 3). In each plot, the positions of the fringes are shifted according to the phase of the input pulse. Note the symmetric fringe movement in the plot of cubic phase, and the nonsymmetric movement for pulses with chirp and quartic phase. The spectrum of the signal from the pulses with phase distortions is not quite as broad as for the flat-phase pulse. The fringe movement is rather subtle. This is in part due to the difficulty of visually discerning fringe movement, but also due to taking care to keep the spectral phase within the validity ranges of SRSI, which will be discussed in the next section.

B. Validity Ranges

Another issue that should be addressed promptly for measurement techniques is the issue of its implicit assumptions and validity range. Many pulse retrieval schemes rely on some assumptions about

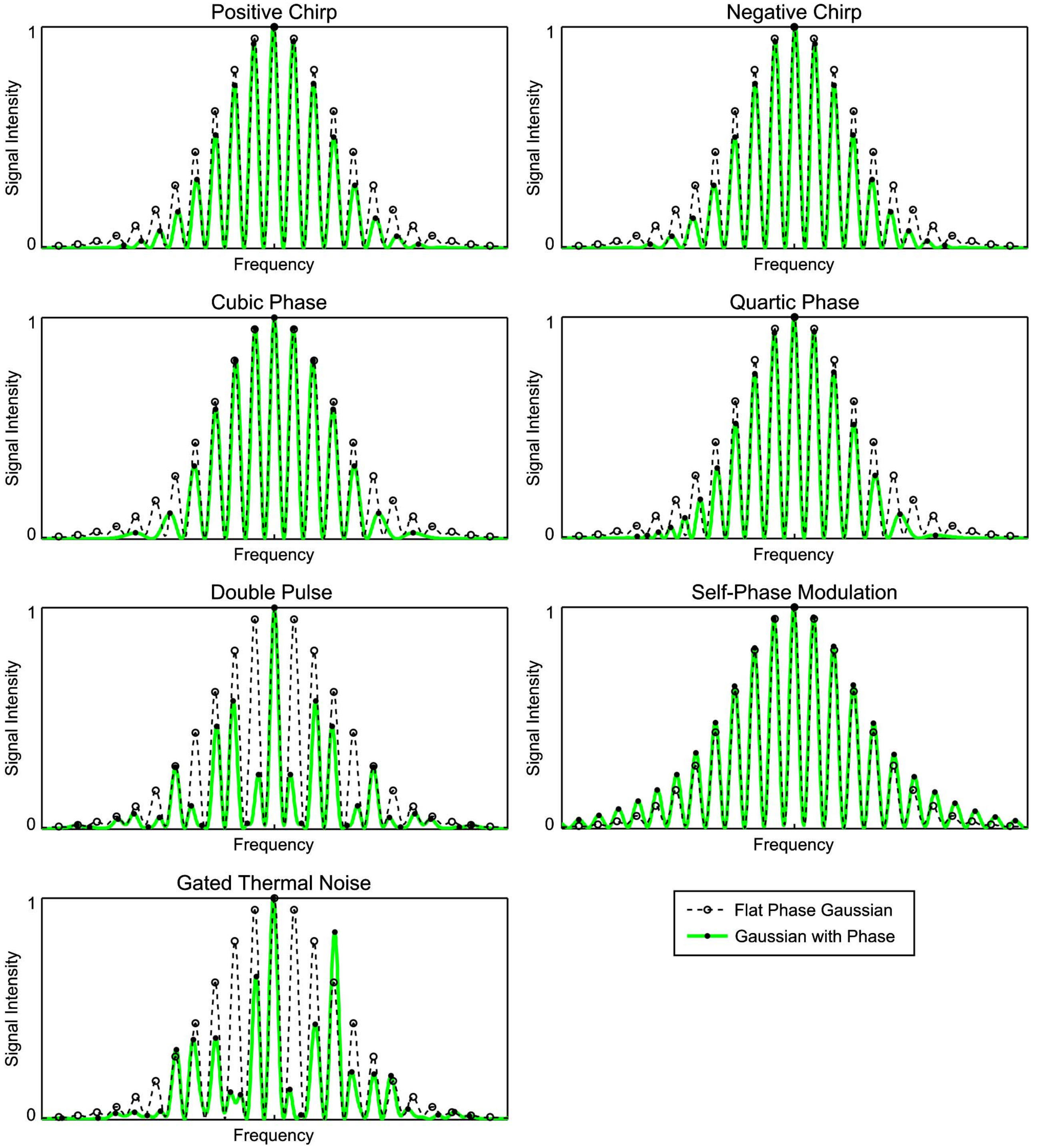

Fig. 3. Example SRSI traces for Gaussian pulses with positive and negative chirp, cubic spectral phase, and quartic spectral phase, as well as a Gaussian double pulse (pulse separation is ~3 times the pulse FWHM), a Gaussian pulse after self-phase modulation, and time-gated thermal noise. For reference, the SRSI trace for a flat-phase Gaussian pulse is also plotted (black dashed line), and the peaks of the fringes of both curves are marked with circles.

the characteristics of the pulse to be measured. This is acceptable in many circumstances; however, the limits of such assumptions should be clearly stated and discussed. If the assumptions required to measure the pulse are too limiting, then it will not be appropriate in many scenarios. As an example, SRSI relies on the input pulse to have a chirp that is small. Otherwise, the reference pulse generated by the XPW process will have a narrower spectrum than the input, resulting in an inability to measure the phase of some of the frequencies present in the input pulse [38]. This restriction is quite limiting, and consequently this method is not endorsed by its creators for pulses chirped to more than twice their

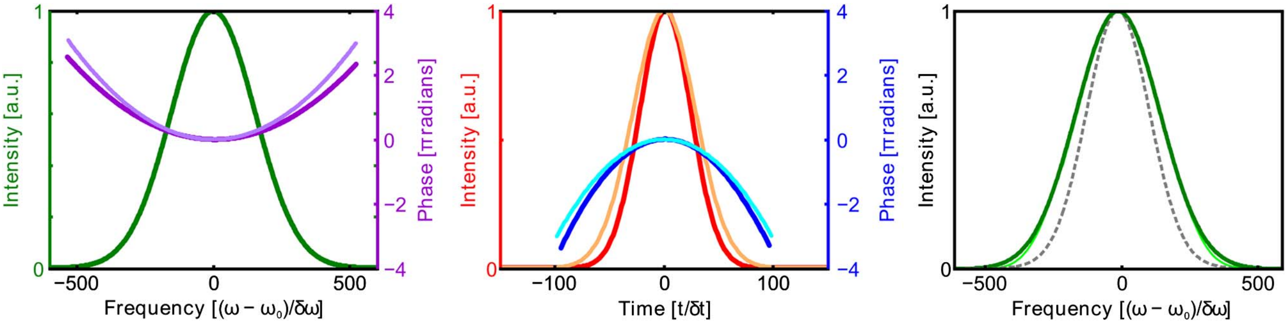

Fig. 4. Example simulated measurement of a pulse that is outside the validity range of SRSI. Left: retrieved spectral intensity (dark green) and phase (dark purple) with actual phase (light purple). Middle: retrieved temporal intensity (red) and phase (dark blue) with actual intensity (orange) and phase (cyan). Right: fundamental (light green thin solid line) and XPW (dotted gray line) spectra retrieved from the trace with independent fundamental spectrum (dark green thick line). The measurement underestimates the chirp of the pulse and its temporal duration.

Fourier-transform limit [36]. And nonideal experimental conditions (such as poor polarizer extinction or alignment) can reduce this validity range even further.

If a pulse is not simple enough to generate a XPW reference with a broader spectrum, then the retrieval process will converge to an incorrect pulse. This is the case for the pulse shown in Fig. 4. While a separately measured input spectrum can be compared with the two measured spectra to indicate an unsuccessful measurement (and to make sure the XPW spectrum has not been incorrectly labeled as the input spectrum), it is difficult to know in advance if this technique will succeed in measuring the pulse. Techniques that can only measure simple pulses are limited in their usefulness, but may have important applications. In these cases, information on how simple the pulse needs to be and what happens when those conditions are not met must be readily available. And it is important to remember that a technique that can only measure simple pulses will always give a simple pulse as a result, and so can yield a simple pulse when the pulse is in fact quite complex.

C. Ambiguities

As many of the problems related to intensity-autocorrelation measurements have been caused by the large degree of ambiguity in the pulse intensity profile [4], it is clear that any ambiguities present in a technique have a large impact on its usefulness. All techniques currently in use have “trivial ambiguities” that are rarely problematic or easily overcome, such as the absolute phase of the pulse, the arrival time of the pulse, and/or the direction of time. But “nontrivial ambiguities,” which are ambiguities that are more difficult to compute or that matter greatly in measurements, should be identified, if possible. Both intensity and interferometric autocorrelation have so many ambiguities that it has not been possible to identify all of them [44].

Very simple ambiguities can be checked analytically, and the relative phases of well-separated pulses or modes have in fact been found to represent ambiguities in most methods [45]. Unfortunately, in general, ambiguities are especially difficult to identify in ultrashort-pulse-measurement techniques because the mathematics of these techniques is inherently nonlinear, making general analytical solutions difficult and generally unavailable. FROG benefits from its equivalence to a well-known, essentially well-posed problem: the two-dimensional phase-retrieval problem. But, in general, identification of ambiguities can only be performed by the brute-force running of large numbers of pulses through a retrieval algorithm. Fortunately, this is not difficult to do, and this has been done for FROG, and, aside from those mentioned above, none have been found [46].

A similar approach yields no nontrivial ambiguities for SRSI within its limited validity range. We simulated measurements of more than 5000 arbitrary pulses and found no cases in which the retrieval converged well to an incorrect pulse, although there were a significant number of pulses for which the retrieval did not converge. Theoretically, while it is clear that no two input pulses should generate the same reference pulse, it has not been proven that a pulse is uniquely determined by the quantities measured by SRSI. These quantities are the input pulse spectrum, the spectrum of the XPW pulse generated from it, and the phase difference between those two pulses. Even though the intensity and phase of the reference pulse is unique for each input pulse, it is not immediately clear that the phase difference is necessarily unique for each set of input pulse and corresponding reference pulse. It seems very unlikely that there would be a large number of nontrivial ambiguities associated with those constraints, however.

D. Coherent Artifacts

In addition, as we have mentioned, it is important to understand how the measurement technique responds to averaging over trains of pulses of varying shape. Since a measurement technique is usually restricted to giving one and only one result, if it actually averages over more than one distinct event, it cannot possibly convey all those events to the user.

This applies to all measurements that use more than one pulse—whether the measurement requires multiple shots or whether a single-shot measurement is averaged over several pulses (as is frequently done to obtain a better signal-to-noise ratio). *All* techniques will be negatively affected by averaging over differing pulses and must necessarily give an incorrect answer. A single-shot *measurement* is generally immune to coherent-artifact effects, but a single-shot *technique*, averaged over several pulses, is not. And although the general trend of the effects of averaging over pulses is most clear when considering an average over a very large number of pulses, these effects are still present and still very problematic when averaging over only a few pulses.

Fortunately, even though it is clearly impossible for the technique to correctly represent all the pulses it measured in a single result, the best that can be expected from a technique in such cases is to give a good representative pulse as well as a clear indication that there is variation in the pulse train. At minimum, an indication of the instability should be present.

The effects of pulse-shape variation are easily simulated by generating pulse trains that have both a stable component that is the same from pulse to pulse and an unstable component that varies. The result of calculating an average measurement over this train should be compared to the result of calculating a measurement of the stable component alone. Even if retrieving a pulse from the averaged unstable measurement gives the same answer as the stable measurement, any feedback or differences in the raw measurements may signal the user that the result does not represent a stable pulse train. While it would be ideal for the measurement to provide a typical pulse, it is absolutely necessary to be able to tell whether the multishot measurement represents a stable or an unstable train.

We have performed these simulations for SRSI using pulse trains containing 5000 pulses, whose unstable components consist of time-gated thermal noise with the same average spectrum as the stable component (see Fig. 5). All three measurements retrieve the same result: a flat-phase pulse that has the same temporal width as the stable component of the pulse train. Thus, SRSI measures only the coherent artifact in an unstable pulse train, as has been found for other pulse-measurement techniques that are based on spectral interferometry [33,34].

The more important question remains, however: are there any significant differences between the

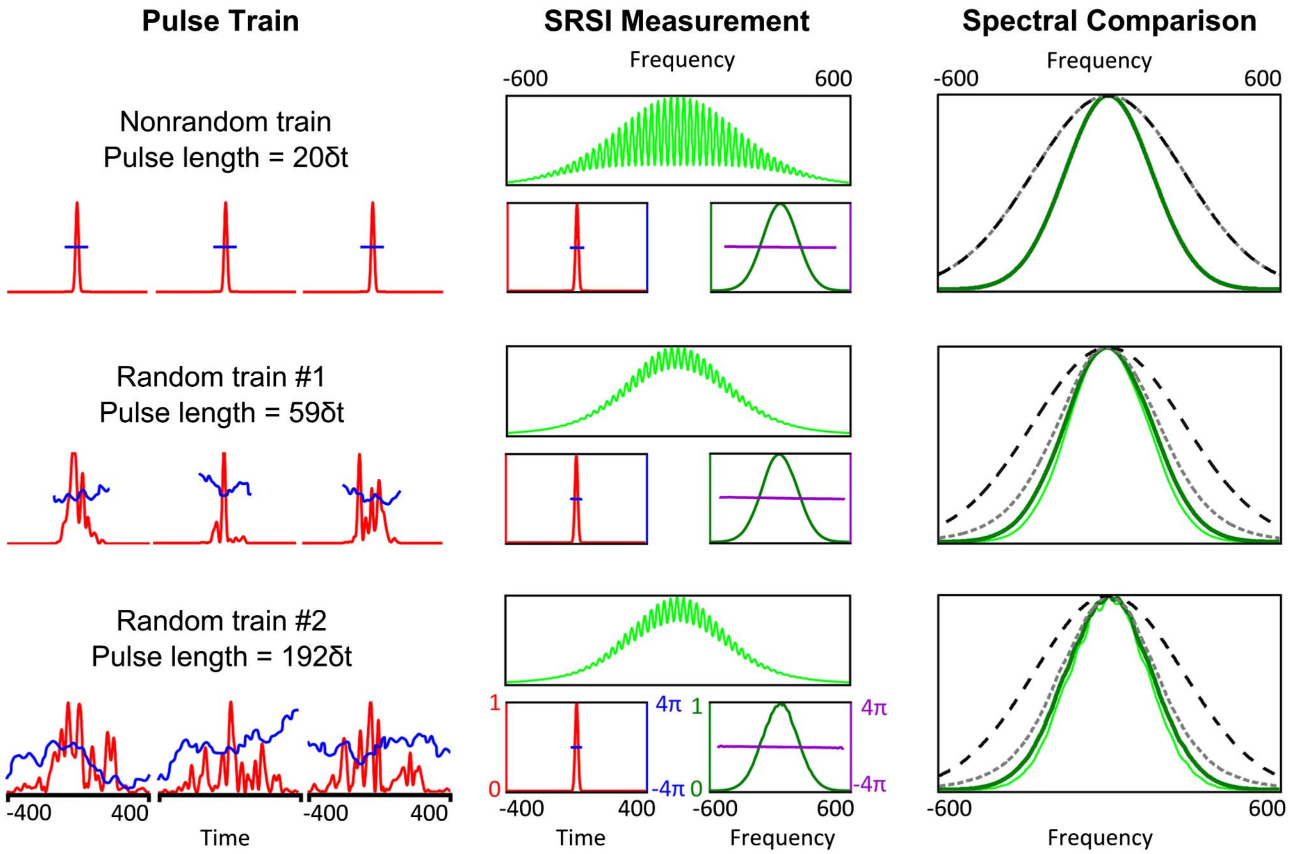

Fig. 5. Coherent-artifact simulation for SRSI. Example pulses are given on the left. The measurement and retrieved temporal and spectral intensity and phase are in the middle column (red is temporal intensity, blue is temporal phase, dark green is spectral intensity, and purple is spectral phase). On the right, four spectra are plotted for comparison: the (average) input spectrum (dark green thick solid line), the spectrum of the input pulse measured in the trace (bright green thin solid line), the spectrum of the XPW pulse measured in the trace (gray dotted line), and the spectrum of the retrieved XPW pulse (black dashed line).

measurements of the unstable trains and the stable pulse that could signal the user that the measurement result is not correct? Contrasting the measurement of a stable pulse with the measurements of unstable pulse trains, we see that for the stable pulse, the spectral intensity measured via FTSI agrees exactly with the input spectrum. Similarly, the retrieved and FTSI measured XPW spectra also agree well for the stable pulse. This is not true for the measurements of the unstable pulse trains. The discrepancy between the measured and retrieved XPW spectra is significant over the whole spectral range for both unstable trains. Since the measured XPW spectrum is still clearly broader than the input spectrum, but not as broad as would be expected from generating an XPW reference from the retrieved pulse, the user can conclude that something is amiss in the measurement. In other words, the retrieved phase is incorrect for reasons other than being outside the validity range of SRSI. Comparing spectra as a feedback mechanism will be discussed in more detail in the next section. In addition to the spectral discrepancies, there is significantly more background present in the traces of unstable trains. To compare these measurements to measurements of the same pulse trains using other methods, please see Ref. [34].

We consider the four issues mentioned above to be both extremely important and very easy to address. As such, we strongly recommend that they be dealt with promptly, ideally in the first paper published on a new measurement technique. There are, of course, a number of other issues that require more effort to investigate, but must be dealt with before the technique should enter common use. The most important of them is feedback and error estimation. In addition, calibration is important, and for iterative retrievals, algorithm convergence conditions should be described.

4. Recommendations

A. Feedback and Errors

An often overlooked component of pulse measurement is feedback. Ideally, there should be some consistency check within the measurement to ensure that the result is not corrupted by unforeseen random or systematic error, failure to obey validity ranges, lack of convergence, or pulse-shape instability. SRSI does have a couple of feedback mechanisms that are quite helpful in guarding against these problems.

As has already been mentioned in the section on validity ranges, an independently measured spectrum should be compared to the two spectra calculated from the trace to make sure that the XPW reference pulse has a spectrum that is at least as broad as that of the input. This is necessary for remaining within the validity range of SRSI. If the input spectrum taken from the trace differs significantly from an independent spectrum, or if neither spectrum calculated via FTSI matches the independent input spectrum, something has gone wrong in the measurement and the measured phase should not be trusted.

In addition, as mentioned in the section on the coherent artifact, the input spectrum and the measured phase difference can be used to iteratively retrieve the input pulse and therefore calculate the XPW reference pulse and its expected spectrum. This can then be compared with the reference XPW spectrum measured in the trace for another, much stronger consistency check. If the measured and retrieved XPW pulse spectra are not similar, again, the measurement result should not be trusted as correct.

To illustrate this, we generated example theoretical measurements of moderately complicated pulses, some of which converged to the correct pulse in the retrieval and some of which did not. We plot the spectral and temporal intensity and phase of these pulses, both actual and retrieved, along with four spectra: the actual spectrum (or independent spectrum), measured spectrum, measured XPW spectrum, and retrieved XPW spectrum. When the measured and retrieved XPW spectra are the same, the retrieved pulse is correct.

Figures 6–8 show these example pulse retrievals. Figure 6 shows a moderately complicated pulse that was not correctly retrieved. This is most evident in the discrepancy in temporal intensity between the retrieved pulse and the actual input pulse. The

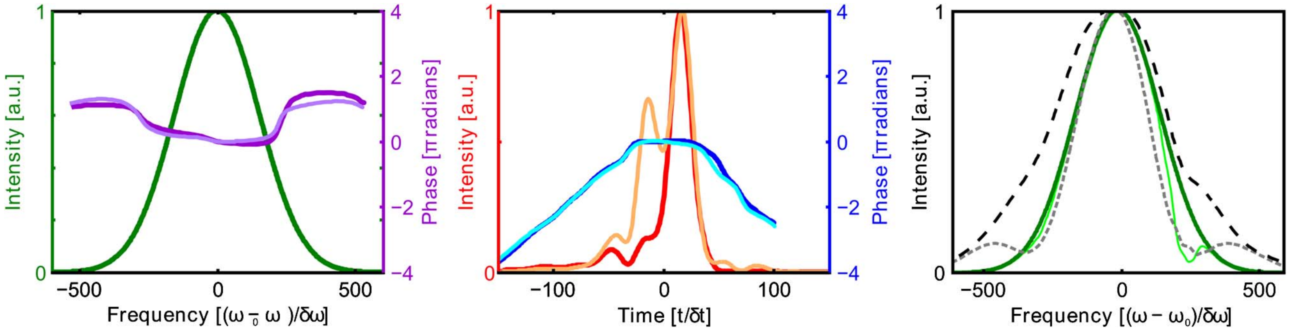

Fig. 6. Example simulated measurement that was not correctly retrieved. Left: retrieved spectral intensity (dark green) and phase (dark purple) with actual phase (light purple). Middle: retrieved temporal intensity (red) and phase (blue) with actual intensity (orange) and phase (cyan). Right: measured input (light green thin solid line) and XPW (gray dotted line) spectra from the trace with independent input spectrum (dark green thick solid line) and retrieved XPW spectrum (black dashed).

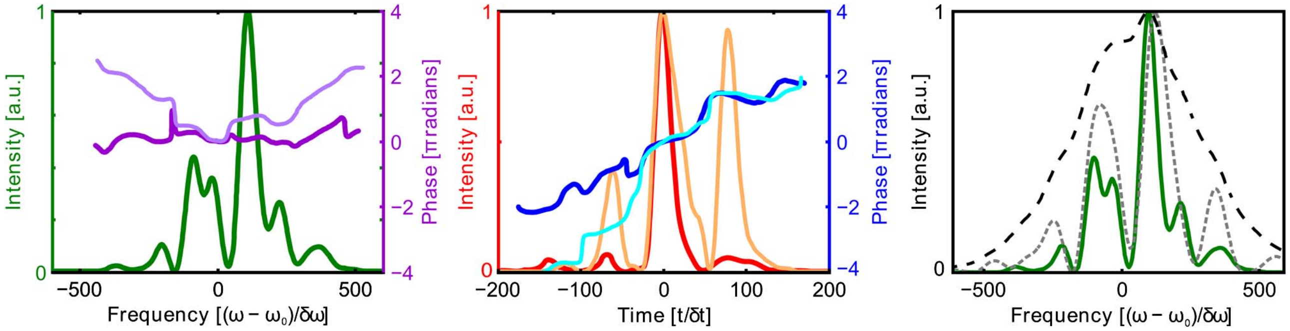

Fig. 7. Example simulated measurement that was not correctly retrieved. See the caption of Fig. 6 for the color key.

retrieved XPW spectrum does not agree with the spectrum measured in the trace, and the XPW spectrum measured in the trace is narrower than the spectrum of the input pulse. This pulse is outside the validity range of the technique, and comparing the spectra makes it easy to identify that this is the issue.

Figure 7 shows a rather complicated pulse that also was not correctly retrieved. Again, the measured XPW spectrum is quite different from the retrieved XPW spectrum, very clearly showing that the retrieval has failed. The XPW spectrum measured in the trace shows that this pulse is on the edge of the validity range, neither obviously broader nor narrower than the input spectrum.

Figure 8 shows a complex pulse that was correctly retrieved. Note the excellent agreement between the input spectrum measured in the trace and the independent spectrum, and likewise between the XPW spectrum measured in the trace and the XPW spectrum simulated in the retrieval.

Contrasting the measured and retrieved XPW spectra plotted in Figs. 6–8 with the corresponding plots in Fig. 5 further reinforces previous conclusions about the simulated measurements of unstable trains. The fact that the measured and retrieved XPW spectra do not match for the unstable pulse trains in Fig. 5 very clearly signals an incorrect measurement. These results suggest that if a pulse appears to be simple and inside the validity range, but it cannot be correctly retrieved, it is likely to be unstable.

In summary, a SRSI measurement is correct if the measured XPW spectrum is broader than the input spectrum and if the measured XPW spectrum matches the retrieved XPW spectrum. Likewise, a measurement that does not have these qualities can be very wrong. Given that such feedback mechanisms exist, they should be used and presented whenever possible to demonstrate that a result is indeed reliable. Without them, there is no way of knowing whether the measurement of an unknown pulse is actually correct or not.

If a quantification of the measurement quality is desired, then one could compute the rms difference between retrieved and simulated XPW spectra (which is sometimes not small at all). No other quantities have been suggested as a metric for measurement quality of unknown pulses measured by SRSI. Many other fields of science commonly use error bars and confidence intervals in reporting their results. Possibly as a legacy of autocorrelation and its many unknowable ambiguities, it is rare in ultrafast optics to see any indication of the degree of certainty in a published pulse-intensity-and-phase measurement. Ideally, one should be able to put confidence intervals on all the measured spectrum and phase points, influenced by spectral agreement and other factors, such as noise. This has been performed using an approach called the bootstrap method, for FROG [47,48]. An analogous approach to that used for FROG for obtaining this information could also work for SRSI measurements.

B. Calibration

All measurements require some number of parameters to turn the raw data into pulse information. The extent of those parameters and the effects of their

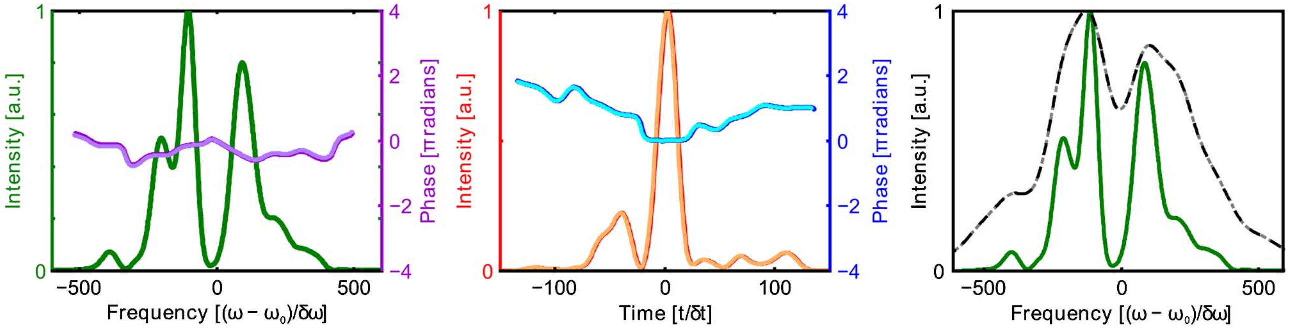

Fig. 8. Example simulated measurement that was correctly retrieved. See the caption of Fig. 6 for the color key.

miscalibration should be described in detail. In particular, the required precision of the calibration and sensitivity of the result to poor calibration is important. As would be expected from a variant of spectral interferometry, SRSI is sensitive to spectrometer calibration. The measurement result can be affected by saturation, variations in frequency response across the spectrum, resolution, and wavelength to frequency conversion. Luckily, these are familiar issues, and schemes for limiting their effect on spectral measurements have been devised [49].

C. Convergence of Iterative Algorithms

In many pulse-measurement schemes, deriving the properties of pulses immediately from measured data is not possible. Instead, the data and the method by which they are created strongly constrain the solution, and the pulse is retrieved by iteratively moving toward the solution that best fits with the measurement. Naturally, the reliability, speed, and convergence conditions of the algorithm to find the solution must be explored.

The SRSI algorithm (described in Section 2) is said to have converged when the modification of the phase on a single pass of the algorithm becomes negligible [41]. Quick convergence is expected when the XPW reference has a spectrum that is at least as broad as the input spectrum. As the input phase approaches the validity limits, the retrieval converges more slowly. It has been shown [41] that for quadratic phase and a Gaussian spectrum, the requirement that the reference spectrum be at least as broad as the input spectrum is a more conservative limit on the convergence conditions than would be imposed by the retrieval algorithm itself. For a Gaussian spectrum, the limits on various degrees of polynomial phase can be calculated, but in general, the convergence conditions of the algorithm must be simulated.

In our limited investigations, we simply ran through 20 iterations of the retrieval algorithm, rather than monitoring the change in the phase on each pass. The pulses that converged did so in less than 20 iterations, and those that failed to converge clearly stagnated well before 20 iterations. Although our simulations were far from exhaustive, in our experience the algorithm for SRSI is quite fast.

5. Conclusion

In conclusion, we argue that there are four issues that would ideally be addressed in the first paper on a new technique: namely, theoretical example measurements, validity ranges, nontrivial ambiguities, and response to unstable pulse trains (the coherent artifact). Beyond that, we strongly recommend that another three issues be addressed before the technique is used to make measurements of significance: feedback, calibration, and algorithm convergence. Because ultrashort pulses are finding such a broad range of applications, especially in medicine, and measuring them is so challenging, it is vital to the progress of ultrafast optics and its applications that pulse-measurement techniques be well understood, along with their limitations and their behavior in a wide range of situations. Our hope is that these guidelines will help researchers quickly identify new, more powerful measurement techniques that show promise and adopt those that work well. In addition, we hope to encourage the development of techniques for ever more reliable and accurate measurements.

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