Basic diffraction phenomena in time domain

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Abstract: Using a recently developed technique (SEA TADPOLE) for easily measuring the complete spatiotemporal electric field of light pulses with micrometer spatial and femtosecond temporal resolution, we directly demonstrate the formation of theo-called boundary diffraction wave and Arago's spot after an aperture, as well as the superluminal propagation of the spot. Our spatiotemporally resolved measurements beautifully confirm the time-domain treatment of diffraction. Also they prove very useful for modern physical optics, especially in micro- and meso-optics, and also significantly aid in the understanding of diffraction phenomena in general.

OCIS codes: (320.5550) Pulses; (320.7100) Ultrafast measurements; (050.1940) Diffraction.

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15. If viewing the plots without magnification in a computer screen the Moiré effect may obscure the actual small period of the intensity oscillations and increase of the period.

1. Introduction

The bending of light waves in the shadow region behind an opaque disk and the appearance of a bright "Spot of Arago" in the shadow centre are well-known manifestations of diffraction. Tremendous progress was made in the mathematical treatment of diffraction in the last two centuries, resulting in the well developed theory with Fresnel-Kirchhoff and Rayleigh-

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Sommerfeld versions. An alternative theory, inspired by the early ideas of Thomas Young, has been developed by Maggi [1], Rubinowicz [2], Miyamoto and Wolf (references given in [3]). The boundary-diffraction wave (BDW) theory, as it was called, describes diffraction from openings in opaque screens in a mathematically simple manner. The BDW theory is especially intuitive when describing the formation of the diffracted field for the case of illumination with ultrashort laser pulses [4].

In the traditional diffraction treatment using monochromatic fields, the transmitted waves fill large depths of space behind the screen and overlap with each other there. In contrast, diffracted ultrashort pulses—typically only a few micrometers "thick"—behave almost like solitary spherical wave-front surfaces emitted from the boundaries of the screen. As a result, for ultrashort pulses, the study of diffraction in terms of pulsed BDW's in the time domain is not only didactically preferable but also opens new interesting directions and applications, such as in the study of focusing and other transformations of ultrashort pulses [5]. The formation of an ultrashort boundary-wave pulse just after a circular aperture has been theoretically studied [4], and experimental evidence for its existence was obtained by measuring modulations in the spectrum of the on-axis field, with CCD-recordings of the time-integrated radial intensity distribution of the field, or using spatial interference [6–8].

In this publication, our aim has been to directly record, with simultaneous spatial and temporal resolution, the evolution and interference of the boundary waves behind an opaque disk and also behind a circular opening. We show that our high temporal resolution reveals similar spots of Arago for both types of screens. It also reveals that the spots are actually decelerating versions of the superluminal Bessel-X pulse (see [9–11] and references therein).

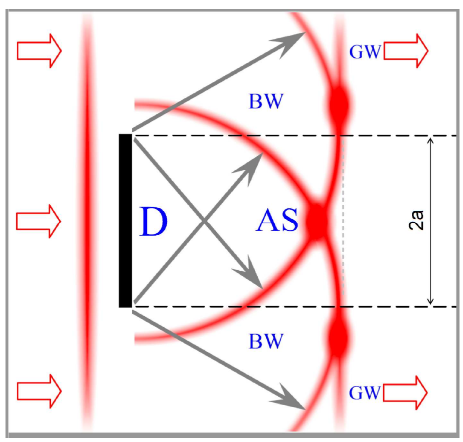

Fig. 1. Schematic of the formation of the Arago spot in the case of illumination with ultrashort pulses. A pancake-shaped pulsed wave from the left illuminates a disk-shaped obstacle (D) with radius *a*. The obstacle removes its central region according to the shadow boundaries (horizontal dashed lines), forming the geometrical-wave (GW) component of the output field. In addition, the edges of the obstacle excite the boundary diffraction wave (BW), which expands from a ring torus shape trough a spindle-torus-like stage (cross-section depicted in the figure) into a spherical wave at infinity. On the axis, overlapping and interfering boundary waves form the Arago spot. Around the shadow boundary in the overlap regions (also indicated by ovals) of the BW and GW, the common interference rings appear. The Arago spot (AS) propagates along the axis behind the front (indicated by vertical dashed line) of the transmitted GW but catches up with the latter at infinity because its velocity is superluminal.

2. Theoretical description of the boundary wave pulse

According to the BDW theory [3] and its modification [4] for plane-wave incident pulses, the field after the diffracting screen consists of two components (see Fig. 1). The first—the so-called geometrical wave—propagates in accordance with geometrical optics, i. e., it is identical to the incident pulse, except in the region of the geometrical shadow, where it is zero. The second component—the BDW pulse—is simply a sum of spherical waves emitted

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along the disk's edge having the temporal profile of the incident pulse. Therefore the BDW pulse for the case of a disk of radius a is given by the wave-function [12]

$$\Psi_{BW}\left(r,z,t\right)\propto e^{-i\omega_0 t}\int_0^{\pi} v\left(t-\frac{s}{c}\right)e^{ik_0 s}\,\frac{ra\cos\varphi-a^2}{\left(z-s\right)s}\,d\varphi, \tag{1}$$

where r and z are, respectively, the radial and axial coordinates of the field point; ω_0 and k_0 are, respectively, the carrier frequency and wave-number, $\omega_0 = c\ k_0$; v is the pulse envelope and $s = (r^2 + z^2 + a^2\text{-}2ra\cos\phi)^{1/2}$. The integration over ϕ stems from contour integration along the disk's boundary and is one-dimensional—this is the known advantage of the BDW theory as compared to common diffraction theories based on 2D surface integrals. The BDW pulse in the case of a circular hole is given also by Eq. (1), but with the opposite sign [4,12]. This could be expected if we recall Babinet's principle. Equation (1) was used for the numerical simulations of diffracted fields, which will be considered below.

3. Experimental results in comparison with simulations

In our measurements, we used a KM Labs Ti:Sa oscillator with 33 nm of bandwidth (FWHM) and an approximately Gaussian spectrum with a central wavelength λ0 = 805 nm. The spot size of the laser beam was 4 mm (FWHM). To perform the complete-spatiotemporal-intensity-and-phase pulse measurements with the required resolution in both space and time, in conjunction with the required sensitivity, we used a technique called scanning SEA TADPOLE (Spatially Encoded Arrangement for Temporal Analysis by Dispersing a Pair of Light E-fields [13,14]) which a variation of spectral interferometry. To make a measurement, we sample a small spatial region of the unknown field with a micrometer sized fiber and then interfere this with a known reference pulse in a spectrometer to reconstruct $E(\lambda)$ for that spatial point. Then to measure the spatial dependence of the field, we simply scan the fiber point by point through the space where the unknown light field propagates, so that $E(\lambda)$ is measured at each position, yielding $E(\lambda,x,z)$. This field can be Fourier transformed to the time domain to yield $E(t,x,z)$. The plots from our SEA TADPOLE measurements, which are shown below, can be viewed as still images or "snapshots in flight," since they are spatiotemporal slices of the magnitude of the electric field $|E(x,z,t)|$ of the pulses. While we could also scan along the y-dimension of the beam, our set up had cylindrical symmetry, so we only scanned along the x-dimension with the fiber at $y = 0$.

First we propagated ultrashort pulses past an opaque disk with a 4mm, generating a hole in the beam. We measured the resulting spatiotemporal field at different distances after the aperture to observe its evolution (Fig. 2). These measurements reveal the spatiotemporal structure of the weak boundary waves and the brighter spot at the center of the beam due to their constructive interference, i. e., the spot of Arago, as it is known in conventional diffraction theory for stationary (monochromatic) fields. Aside from the noise in the experimental results they are in good agreement with the simulations. The only discrepancy is in the spatial intensity profile of the GW pulse. This is because our beam's spatial profile was Gaussian in the experiments, but to simplify the simulations we used a plane wave spatial profile (see Eq. (1)). In the measured spatiotemporal intensities shown in Fig. 2, two interference patterns are seen. The one with a broader spatial modulation consisting of rings at the edge of the direct (GW) pulse is due to the GW overlapping and interfering with the BW pulse. Another with a much higher spatial frequency—which decreases with the propagation distance [15]—can be seen around the bright central spot and is due to the interfering boundary waves. This part of the field is analogous to a Bessel-X pulse.

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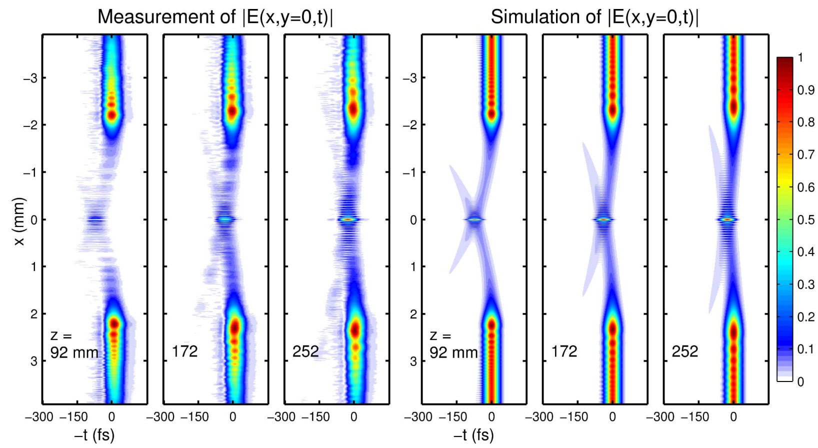


Fig. 2. Formation and evolution of the Arago spot behind an opaque disk 4mm in diameter. The magnitude of the electric field E is shown at three different propagation distances z in pseudo-color code according to the color bar (white has been taken for the zero of the scale in order to better reveal areas of weak field).

In the next measurement we propagated the beam through a circular hole with a diameter of 4.4mm (Fig. 3), which is almost the same as the disk's diameter. The temporal localization of the pulse and our high temporal resolution allows the BW pulse with its central spot to be separated from the direct pulse. If longer pulses or steady-state illumination were used, these two contributions to the diffracted field would overlap and be indistinguishable from one another. These measurements make it clear that the so-called spot of "Arago" is also present due to diffraction from an aperture. This is expected considering that both the aperture and disk have the same boundaries, and should therefore also have the same boundary waves.

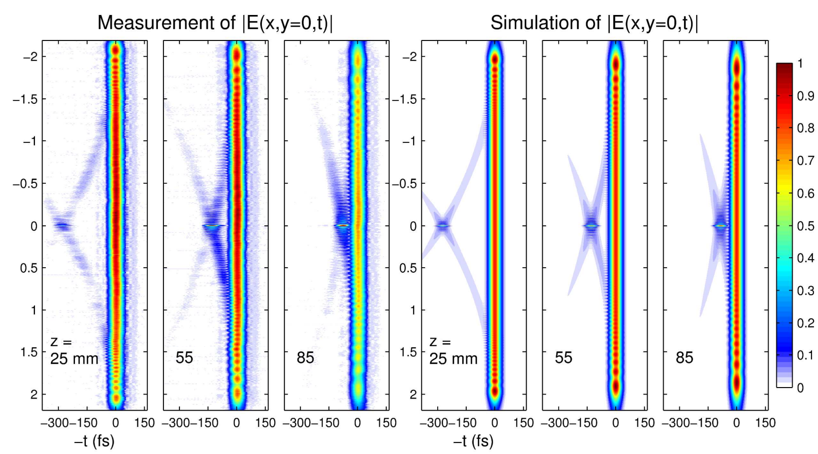
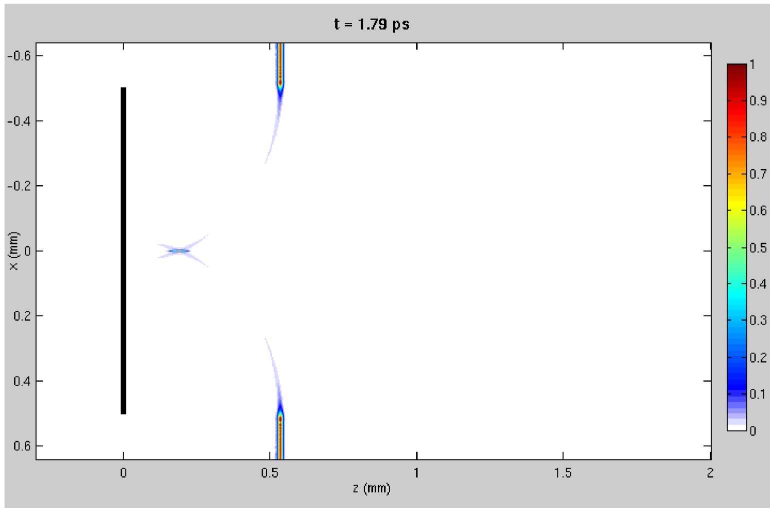
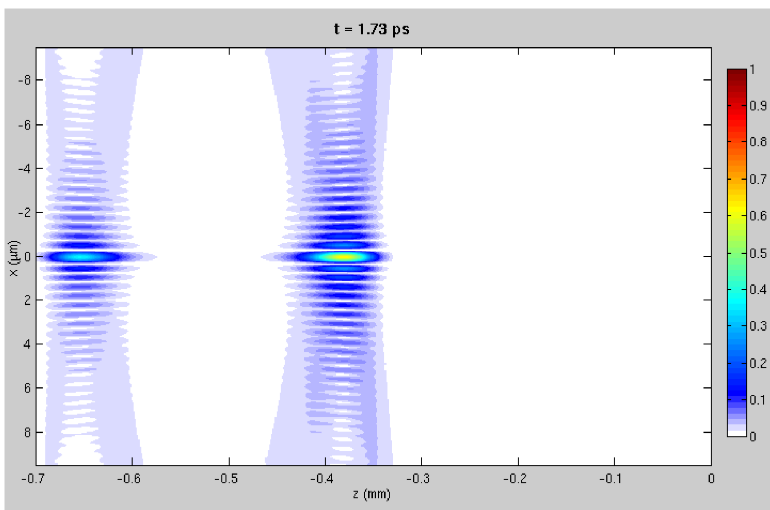


Fig. 3. Formation and evolution of the diffracted field behind a circular hole 4mm in diameter. The boundary waves interfere with each other and with the directly transmitted pulse, but the interference maximum on the axis (actually a temporally resolved spot of Arago) lags behind the direct pulse, and eventually catches up with it.

Interestingly, the plots for both screens reveal that the spot of Arago is surrounded by coaxial interference rings which, in the axial region the field, almost exactly follow the Bessel (J_0) radial profile. Moreover, the spot is delayed in time with respect to the main pulse front, and this delay decreases with z, indicating a superluminal propagation speed along the z axis (the GW pulse front propagates at c). This occurs, because, as z (the distance from the screen) increases, the extra distance that the boundary waves must propagate (compared to the GW pulse front) to reach the z axis ($x = 0$) decreases, so the relative delay of the boundary waves on the axis decreases. As a result, the axial group velocity of the Arago spot—geometrically located at one pole of a luminally expanding spindle torus formed by the boundary diffraction wave pulse—varies from infinity at $z = 0$ to c for very large values of z. Therefore, the spot of Arago is in fact just a decelerating superluminal Bessel pulse like that recently generated using compound refractive optical elements and also studied with SEA TADPOLE [11].



(a)



(b)

Fig. 4. Videos showing simulations of the diffraction of a plane wave pulse from a circular disc of $d = 1$ mm diameter. The pulse parameters are the same as in Figs. 2 and 3. Color represents the normalized amplitude of the electric field. (a) The reference frame is fixed with respect to the disc at $z = 0$ (Media 1). The diffracted field is calculated for $z > 0$ mm. (b) Close-up of the evolution of the boundary wave pulse in a reference frame moving at the luminal velocity c, or with the incident plane wave pulse (Media 2). Note that the x-scale in (b) is finer than in (a) by two orders of magnitude.

Naturally, there is nothing startling about the superluminal speed of the spot, because it cannot carry any information superluminally. This is because it is reformed at every point along its propagation axis (the z-axis) by the expanding spherical-wave constituents, which travel at an angle with respect to the z-axis. It is important to realize that the central

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interference region of the BDW pulse is not moving at this tilt angle. Its phase (and pulse) fronts are perpendicular to the z-axis and move along this axis. Its Poynting vector, indicating the direction of energy flow, also lies along the z-axis. However, the energy flux is not superluminal. The superluminal pulse's velocity should not, of course, be confused with the signal velocity. As is well known, Maxwell's equations, or the wave equation for electromagnetic fields, does not allow superluminal signaling.

To further study the formation and evolution of the boundary-wave pulse, the propagation of an ultrashort pulse was computed in detail behind an opaque disk 1mm in diameter. In the simulations a smaller disk diameter was used in order to reveal the subtleties of the formation of the boundary wave pulse. Figure 4(a) (Media 1) shows the incident and diffracted pulse propagation in the laboratory reference frame where z = 0mm is the location of the disc. The simulations show the creation of the Arago spot at the moment when the expanding ring torus of the boundary-wave pulse becomes an expanding spindle torus. In the close vicinity after the disc, speeds much greater than c can be seen, where the boundary wave pulse literally jumps out of it. Due to the discrete color scale of the animations, the expanding spindle torus shape of the BDW pulse seems discontinuous during the first few picoseconds of the spot evolution, albeit it is only low in intensity in these particular directions.

The Bessel-like radial pattern is depicted in greater detail in Fig. 4(b) (Media 2) where only the field near the axis' center is shown. This time we use a reference frame that is moving with the incident plane-wave pulse. The origin of the frame z = 0mm is bound to the plane-wave pulse moving at velocity c to the right. The fringe pattern in the axial region of the boundary-wave pulse stretches during the propagation as the angle of intersection between the elementary wavelets decreases continuously. Correspondingly, the pulse velocity decreases as the fringe periodicity increases. Since the expansion rate of the spindle-torus is constant (equal to c), the spot on the axis propagates superluminally, decelerating toward the limiting value c. Interestingly, the first seconds of the video also reveal the back-diffracted pulse, which follows from the direct evaluation of Eq. (1). This backward propagating contribution is expected since the spherical waves generated at the boundary of the disk are emitted at all angles in the x-z plane. Of course, the intensity of the backward-moving pulse quickly decreases towards negative values of z and practically ceases to exist within the first millimeter of propagation.

4. Conclusions

In summary, we have performed direct spatiotemporally resolved measurements of pulsed light fields behind basic types of diffracting screens and have interpreted the results using the boundary diffraction wave theory. The latter provides a one-dimensional integral expression for the diffracted field, which enabled us in a computationally simple way to simulate the evolution of the diffracted field. We believe that time-resolved measurements and a time-domain treatment of diffracting waves not only turn out to be fruitful in modern physical optics, especially in micro- and meso-optics, but also promote the understanding of diffraction phenomena.

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