Beyond error bars: Understanding uncertainty in ultrashort-pulse frequency-resolved-optical-gating measurements in the presence of ambiguity

Ziyang Wang, Erik Zeek, Rick Trebino

School of Physics, Georgia Tech, Atlanta GA 30332 USA

Paul Kvam

School of Industrial and Systems Engineering, Georgia Tech, Atlanta, GA 30332

wangziy2000@yahoo.com

http://www.physics.gatech.edu/gcuo/

Abstract: Previously we showed that the "bootstrap method," a well-known statistical technique, can be used to automatically compute error bars in ultrashort pulse measurements using frequency-resolved optical gating (FROG) without the need for additional measurements or traditional error analysis. Here we extend the bootstrap method to pulse measurement in the presence of ambiguities, where traditional error bars would give misleading information. As a result, we provide a new approach to displaying this uncertainty, which nicely reveals the richness of information available (or perhaps unavailable) in a FROG trace (or other measurement) in the presence of ambiguities.

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OCIS codes: (320.7100) Ultrafast measurements; (320.7160) Ultrafast technology; (320.5550) Pulses

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References and links

1. R. Trebino, *Frequency-Resolved Optical Gating: The Measurement of Ultrashort Laser Pulses* (Kluwer Academic Publishers, Boston, 2002).
2. Jung-Ho Chung and Andrew M. Weiner, "Ambiguity of Ultrashort Pulse Shapes Retrieved from the Intensity Autocorrelation and the Power Spectrum," J. Special Top. Quantum Electron. **7,** 656-666 (2001).
3. Y. M. Bruck and L. G. Sodin, "On the ambiguity of the image reconstruction problem," Opt. Commun. **30,** 304–308 (1979).
4. Z. Wang, E. Zeek, R. Trebino, P. Kvam, "Determining error bars in measurements of ultrashort laser pulses," J. Opt. Soc. Am B **20**, 2400-2405 (2003).
5. R. P. Millane, "Phase retrieval in crystallography and optics," J. Opt.Soc. Am. A. **7,** 394–411 (1990).
6. A. Walther, "The question of phase retrieval in optics," Opt. Acta. **10,** 1–49 (1963).
7. Stark, H., ed. *Image Recovery: Theory and Application*. (Academic Press, Orlando, 1987).
8. Brad. Efron and R. J. Tibshirani, *An Introduction to the Bootstrap* (Chapman & Hall/CRC, Boca Raton, 1993).
9. C. Davison and D. V. Hinkley, *Bootstrap Methods and Their Application* (Cambridge University Press, Cambridge, 1997).
10. D. Keusters, H.S. Tan, P. O'Shea, E. Zeek, R. Trebino, W. S.Warren, "Relative-Phase Ambiguities in Measurements of Ultrashort Pulses with Well-Separated Multiple Frequency Components," J. Opt. Soc. Am B **20**, 2226-2237 (2003).

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1. Introduction

For many years, the only available measure of an ultrashort laser pulse was the autocorrelation. Unfortunately, an autocorrelation typically corresponds to many—often quite different—intensities, so even a perfect noise-free measurement of the autocorrelation results in a large and unknown uncertainty in the pulse's intensity vs. time.[1-3] As a result, it is not possible to determine a pulse intensity (or phase) from an autocorrelation, and it makes no sense to attempt to place error bars on such reconstructions of the pulse's intensity (or phase).

Of course, now, methods such as frequency-resolved optical gating (FROG)[1] retrieve a pulse's full intensity and phase vs. time without assumptions. And we recently showed[4] that it makes sense to place error bars on the intensity and phase retrieved in a FROG measurement, which we accomplished using a well-known statistical method, the "bootstrap technique." The bootstrap technique applied to FROG involves running the FROG retrieval algorithm (a modified phase-retrieval[5-7] algorithm) numerous times for a given trace, but on a resampled data set (randomly chosen data subset) from that trace each time. The measured intensity and phase vs. time (or frequency) are then the means of the retrieved intensities and phases for each time or frequency. The error bars are the standard deviations of those values.

As is the case for all ultrashort-pulse measurement techniques, however, there are a few ambiguities in FROG measurements. FROG does not determine the pulse height, absolute phase, or arrival time. While these ambiguities are often actually desirable (for example, it is rarely of interest to measure the distance from the laser to the measurement device, and stabilizing it would be quite inconvenient), and they are usually referred to as "trivial ambiguities,"[1,5-7] as most researchers can live with them, they nevertheless pose problems in implementing the bootstrap method. The absolute phase ambiguity, for example, adds the same arbitrary constant phase to each retrieved phase in the bootstrap run, which, if not controlled, would cause the width of all phase error bars to approach 2π. The pulse arrival-time ambiguity would analogously yield erroneously large intensity error bars. In our previous publication,[4] we showed that these ambiguities can be removed by requiring all bootstrap intensity-and-phase solutions to have $I(0) = 1$, $\phi(0) = 0$, and a pulse intensity center of mass of zero, where $I(t)$ is the intensity vs. time and $\phi(t)$ is the temporal phase. Dealing with the trivial ambiguities in this manner proved quite effective, and we will continue to use this approach for these trivial ambiguities in this more general treatment of error.

There are, however, also *nontrivial* ambiguities in FROG (and all other pulse-measurement methods). These include the relative phases of well-separated pulses in time and frequency in some versions of FROG. These also include an ambiguity in the direction of time in the second-harmonic-generation (SHG) version of FROG, whose traces are necessarily symmetrical in delay and hence cannot distinguish a pulse from its time-reversed replica. In terms of the complex amplitudes, this means that $E(t)$ and $E^*(-t)$ are both possible phases. This latter ambiguity is a nontrivial ambiguity and cannot be reliably removed using a normalization procedure, as described above.

Applying the error bar/bootstrap method as discussed in our previous publication[4] in the presence of the direction-of-time ambiguity yields inappropriate error bars. Figure 1 shows that, for a given SHG FROG trace, the parabolic phase could have a positive or negative leading coefficient (that is, the parabola could "hold" or "spill" water). In the presence of this ambiguity, the bootstrap method, as previously described, would yield a nearly flat phase with increasingly large error bars as time (or frequency) approaches the plot edges. This is in strong contrast to our knowledge of the pulse phase from the measurement, which is that it is quite accurately either one parabola or the other, and definitely not flat.
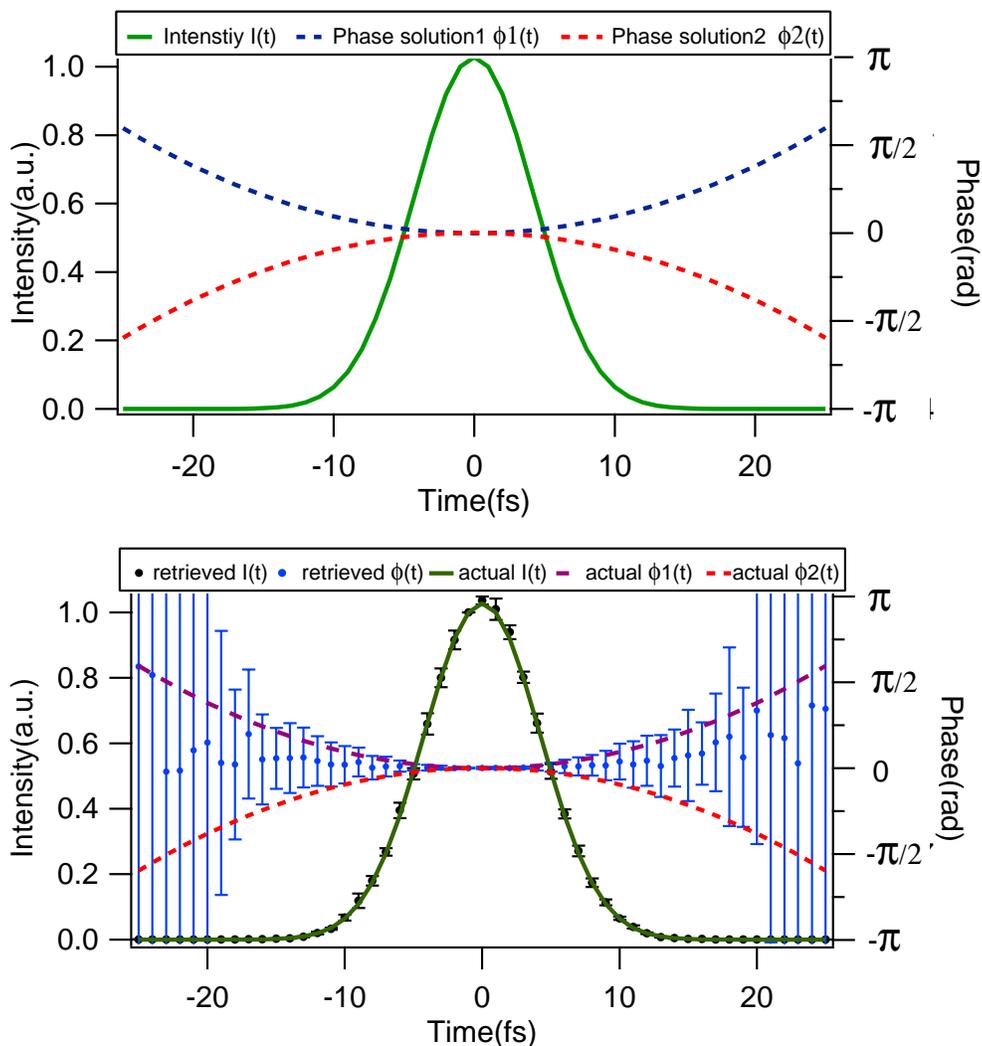
Fig. 1. (a). The intensity (green curve) and the two possible phase solutions (red and blue dashed curves) in an SHG FROG measurement of a linearly chirped pulse. Even in the absence of noise in the trace, half the bootstrap retrieved pulses would yield one phase solution and half would yield the other. Of course, only one is correct. (b). The retrieved intensity and phase using the bootstrap method for the same pulse (in the presence of 1% additive noise, although this is not important). Note that both the retrieved phase and its error bars are unacceptable, giving the impression that the most likely phase is approximately flat with increasingly large errors near the plot edges, rather than the correct result that the phase is quite accurately one parabola or the other.

While it is conceivable that we could remove the direction-of-time ambiguity and other nontrivial ambiguities as we have the trivial ambiguities, a *general* method for dealing with ambiguities would be preferable. Indeed, it is possible that there are other, as yet undiscovered, ambiguities, and we'd like to account for them in advance even if we do not know them.

We solve this problem by realizing that error bars are actually *not* the appropriate approach in this case, where the distribution of possible values is *double-peaked*. Instead, we point out here that the bootstrap method in its most general form yields, not only error bars, but also the entire probability distribution for each value.[8,9] With this observation, we show that plotting *all* of the retrieved intensity-and-phase solutions obtained from the bootstrap

technique nicely reveals the true solution and its potential ambiguities. Indeed, it also reveals the true probability distribution of the measured intensity and phase at each point and is the most general display of the rich information available for the pulse from the data. The remainder of this paper describes the details of the implementation of this more general approach to error in FROG measurements of ultrashort pulses.

2. The bootstrap method

In general, the bootstrap method[8,9] involves taking the data set of M points, randomly resampling a new set of M points from it, one at a time—but *with replacement* (resulting in some points occurring more than once and others not at all)—and then running the relevant parameter-determination algorithm on this new set of points. If this process is repeated numerous times on a given data set, each desired parameter will be determined numerous times. This newly created data set of bootstrap estimates, which can be arbitrarily large, determines the estimated uncertainty and corresponding error bars for the determined parameters.[8,9] In particular, the mean and standard deviation of each parameter's distribution are the measured parameter and the error bar, respectively, when the distribution is normal, or at least uni-modal. This method is called the bootstrap method because it appears that one is getting something for nothing ("pulling oneself up by his own bootstraps"), but this is not the case, and, in fact, the method can be quite computation-intensive. With the use of modern computers, however, the required computations are not prohibitive, and the computations we perform require at most a few minutes on today's personal computers because the FROG algorithm requires as little as 30 ms to retrieve a pulse from a 64 x 64 trace and no more than a few seconds for complex pulses.

Applying this approach to ultrashort-pulse measurement simply involves running the FROG retrieval algorithm on the order of 100 times on the measured FROG trace, but each time with a bootstrap set of points. To choose the bootstrap set of points, we take the original, measured FROG trace and select at random, but with replacement, a number of points equal to the original number, as described above. This allows data points to be selected more than once, and yielding about 2/3 of the points represented in this new trace. Data points not selected are simply ignored in the FROG algorithm (in the magnitude replacement step). Running the algorithm with only a fraction of the points does not harm its accuracy. Indeed, the degree to which the solution varies when points are removed reflects the desired measure of the error. And, as mentioned above, it has been shown that, in general, using this procedure, the statistics of the retrieved values accurately approximate the actual statistics of the derived parameters[8,9]—in this case, the intensity and phase values at the various times (and frequencies). When the distribution is near normal, the mean intensity and phase values for each time and frequency are then the measured values, and the standard deviations yield the error bars. This works because the FROG trace over-determines the pulse, that is, contains many more points than the resulting intensity and phase. In the presence of ambiguity, however, the distribution of values will no longer be near-normal, and knowing the entire bootstrap distribution will then be essential for understanding the uncertainty in the measurement.

3. Double-peaked bootstrap distributions

The problem with the assignment of error bars in the presence of ambiguities is that the bootstrap distribution of retrieved values is far from normal and is instead bi- or multiple-modal for each value of time or frequency (or flat in the case of a completely undetermined parameter). The above bootstrap computation of error bars (standard deviations) of our previous publication is only appropriate when the distribution is near-normal, or at least uni-modal.

Fortunately, as mentioned, the bootstrap method yields the full probability distribution of retrieved values and clearly reveals the bi-modal distribution of phase values in, for example, the direction-of-time ambiguity in SHG FROG. Figure 2 shows a plot of several of the

bootstrap solutions obtained for the pulse in Fig. 1. Note that the two possible phase parabolas emerge, with essentially no values in between. (Do not be distracted by another effect: the fact that the phase becomes random in the wings of the pulse when the intensity goes to zero, reflecting the phase's fundamental indeterminacy for near-zero intensities.) This plot much more accurately reveals the distribution of retrieved values and the error distribution. Thus, when ambiguity is present, it is important to show, not just error bars, but the complete distribution of retrieved values, that is, the complete set of bootstrap solutions. Plotting all bootstrap solutions reveals the true distribution of values and yields a far superior picture of the information in the measurement. (It also nicely reveals the phase indeterminacy when the intensity is zero.)
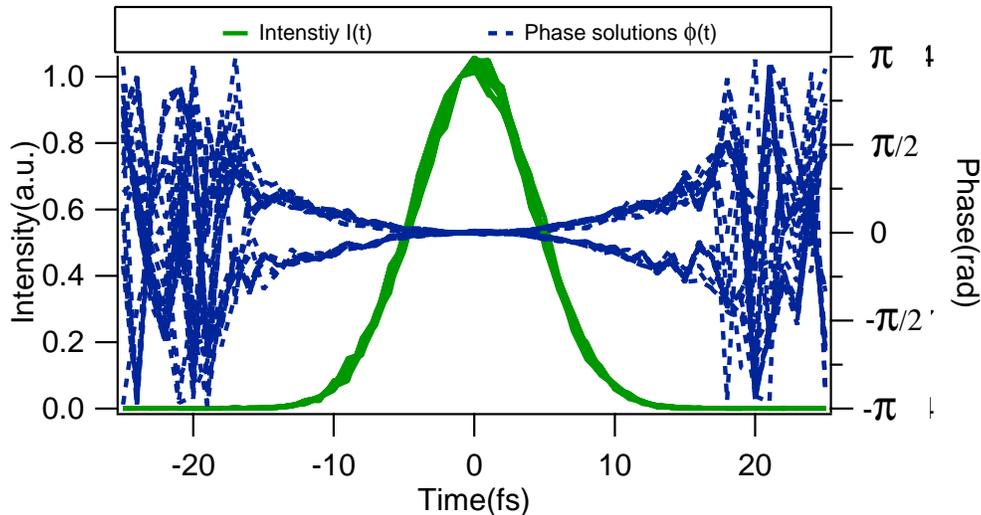

Fig. 2. The entire set of bootstrap solutions for the linearly chirped pulse in Fig. 1. Note that this display much better reveals the true uncertainty in the measured intensity and phase.

When such ambiguous solutions overlap at a point, however, simply plotting all solutions may not remove all sources of confusion. For example, for the above linearly chirped pulse, a source of confusion remains. The ambiguous solutions cross at $t = 0$, and it is not clear, simply by looking at Fig. 2, whether the negative-going solution at negative times continues to be negative-going for positive values of time or whether it becomes positive-going. In other words, the plot could be consistent with saddle-shaped curves in addition to parabolas (see Movie 1, representative frame Fig. 3.). Thus, it is helpful to use different colors or a movie of the various bootstrap solutions to portray the bootstrap solutions. Movie 2 (representative frame Fig. 4.) shows the various solutions for the linearly chirped pulse, much better revealing both the ambiguity and noise.
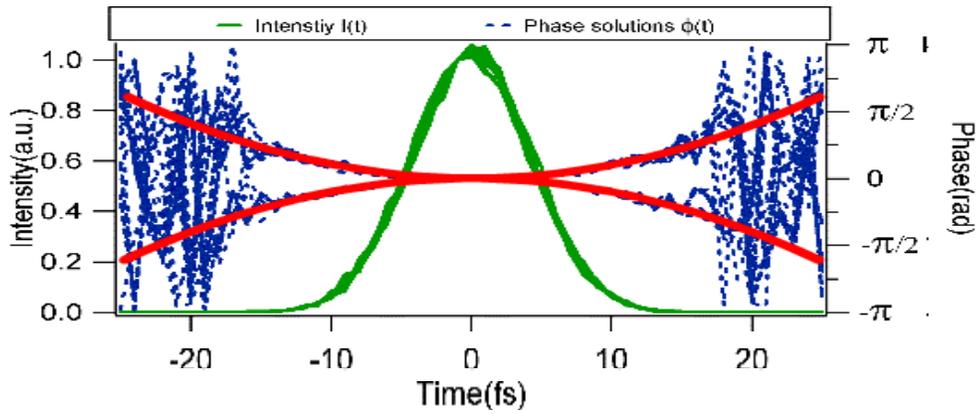
Fig. 3. (1.3MB) Movie of two incorrect saddle-shaped phase curves that could be mistaken for the actual parabolic phase in Fig. 2. Such confusion occurs any ambiguous curves intersect.
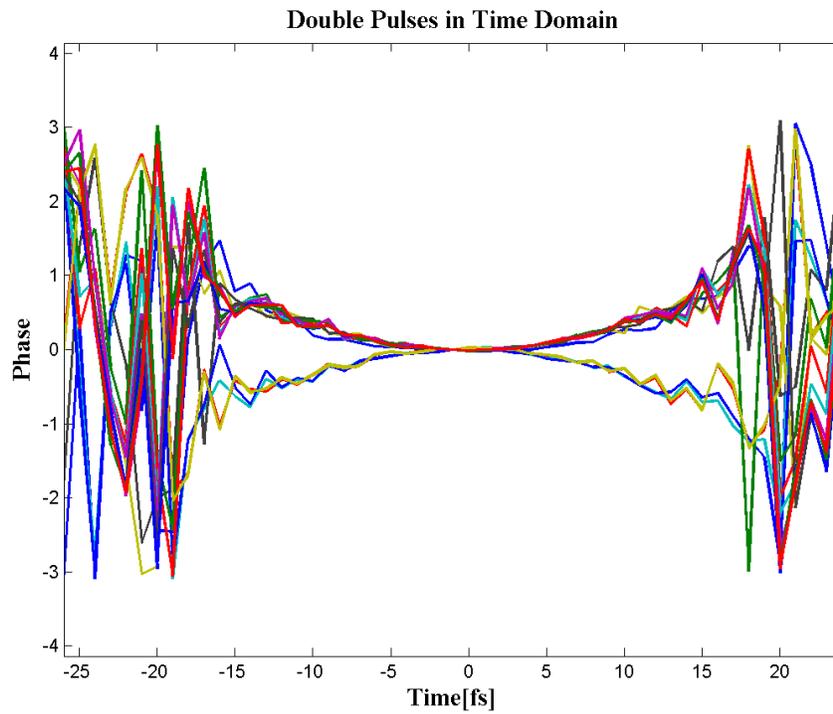
Double Pulses in Time Domain

Fig. 4. (0.9MB) Movie of bootstrap solutions of linear chirped pulse from SHG FROG trace. It clearly reveals the ambiguity and noise of the result.

4. Application of the bootstrap method to other cases of ambiguity

To illustrate the use of this more general bootstrap approach to displaying error, we consider some additional cases of ambiguity.
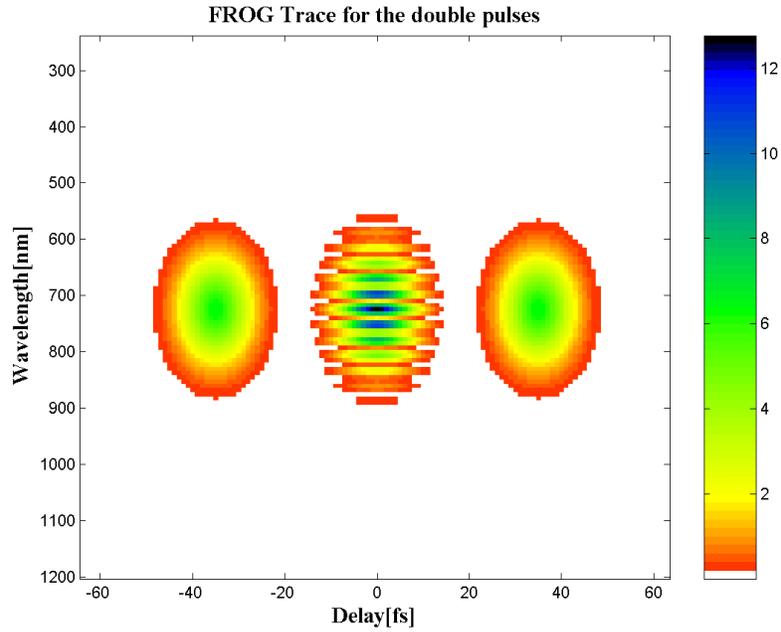

Fig. 5. SHG FROG trace for a double-peaked pulse with a relative phase of π between two peaks.
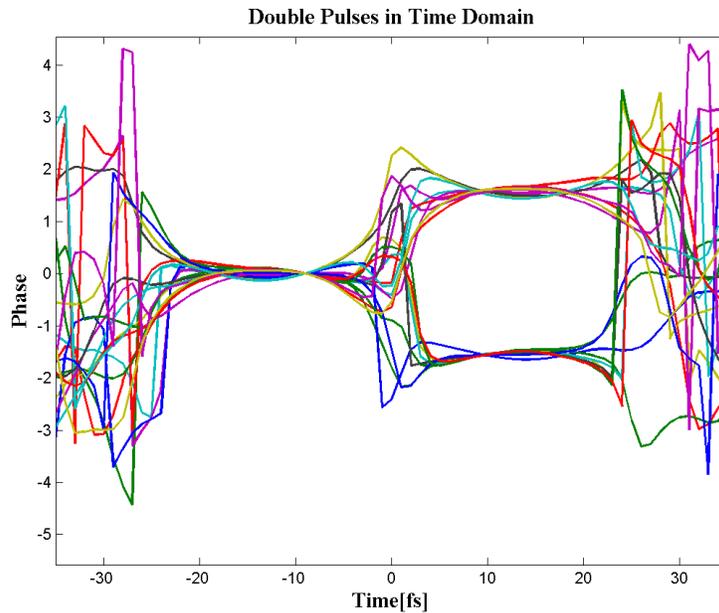

Fig. 6. (0.6MB) Movie of bootstrap solutions for the well-separated doubled-peaked pulse in time. In the plot, the first pulse's phase is set to zero for all pulses.

First, we consider the case of well-separated pulses in time, whose relative phase is ϕ_0, but which, in SHG FROG, cannot be distinguished from another relative-phase value, $\phi_0 + \pi$. Figure 5 shows the SHG FROG trace for such a pulse, and Movie 3 (representative frame Fig. 6.) shows the various bootstrap solutions obtained in this case.

In addition, well-separated frequency components have undetermined relative phases (in FROG and all other pulse-measurement techniques used today, except for XFROG with an appropriately chosen reference pulse).[10] Movie 4 (representative frame Fig. 7.) shows the bootstrap solutions for polarization-gate (PG) version of FROG trace in this case.

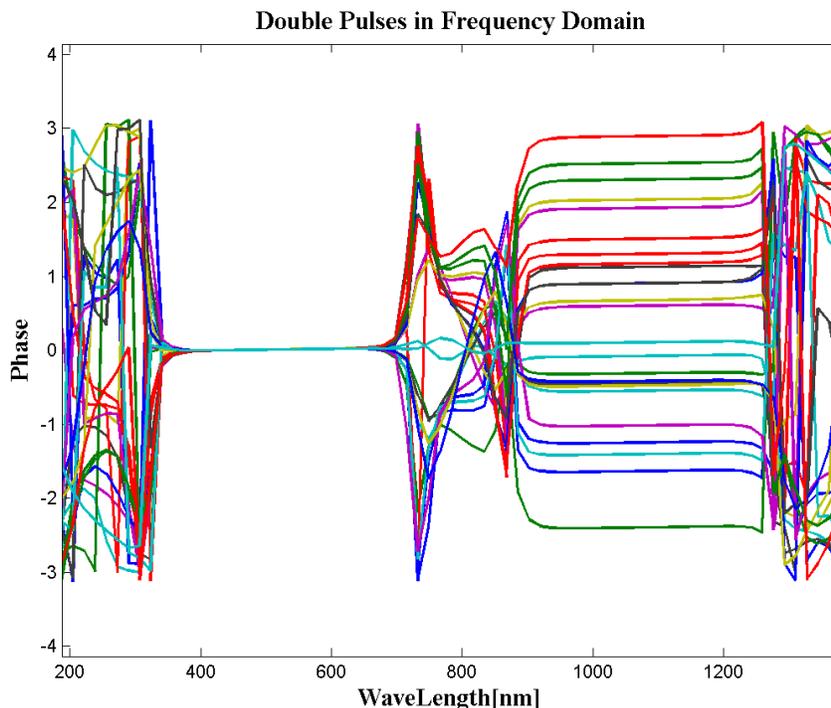**Double Pulses in Frequency Domain**

Fig. 7. (0.9MB) Movie of the bootstrap solutions of a pulse with well-separated frequency components. In this plot the phase of the first spectral component was set to zero for all retrieved pulses.

Another interesting case is the possibility of "approximate ambiguities." Approximate ambiguities have not previously been discussed in the literature, but we define them to be additional, qualitatively different solutions that are consistent with the data in the presence of noise, but which are easily ruled out in the absence of noise. Approximate ambiguities will also reveal themselves by a double-peaked distribution in the bootstrap solution set in the presence of noise.

Here we show that an approximate ambiguity is represented by the case of two "somewhat separated" frequency components, whose intensities overlap enough to prevent ambiguity in their relative phase in the absence of noise. But when some (here 1% additive) noise is added to the FROG trace, the well-separated frequency-component-phase ambiguity appears. Figure 8 shows the bootstrap solutions for the noise-free and noisy traces.

Seperated Frequency Components in the Absence of Noise

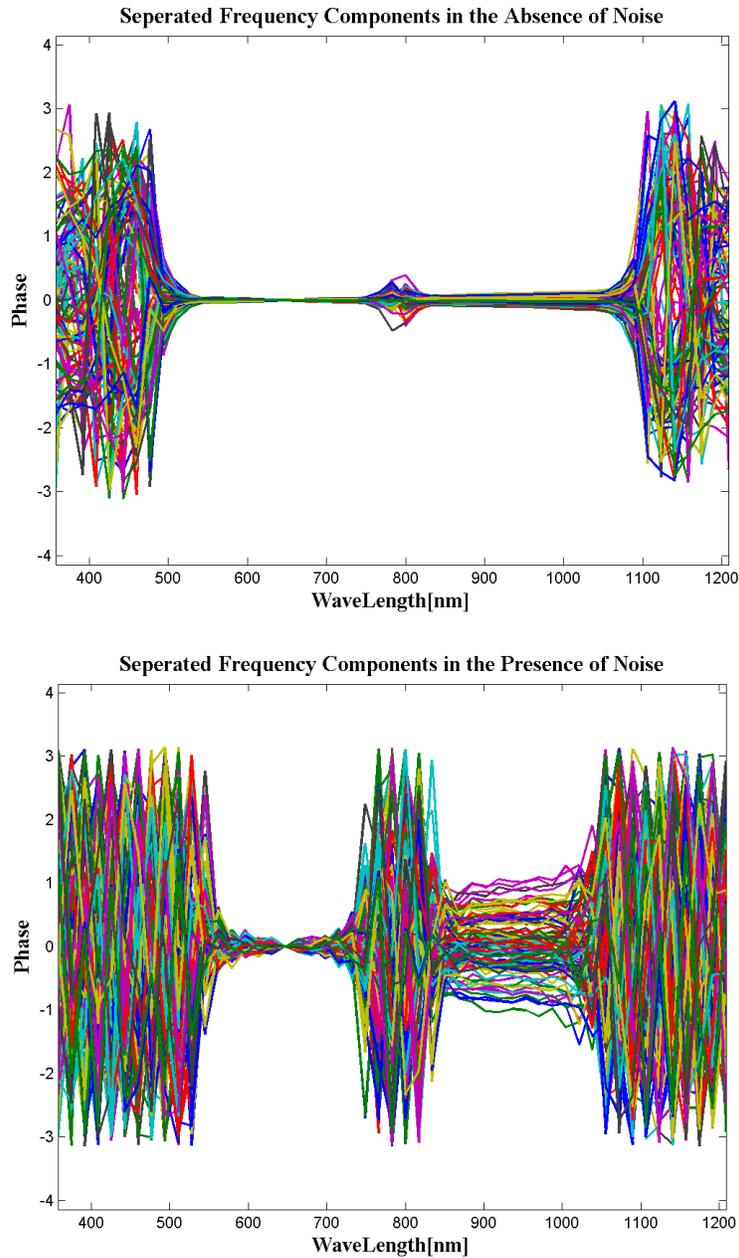**Seperated Frequency Components in the Presence of Noise**

Fig. 8 (a). Bootstrap solutions for a noise-free FROG trace for a pulse with somewhat separated spectral components. Note that the solutions accurately determine the relative phase of the frequency components (although some uncertainty is beginning to appear in the phase of the second component). (b). Bootstrap solutions for a FROG trace for a pulse with somewhat separated spectral components, here with 1% additive noise added to the trace. Note that the solutions no longer accurately determine the relative phase of the frequency components (although the uncertainty is not yet 2π).

5. Conclusions

The bootstrap method allows us to characterize the uncertainty in the measured pulse intensity and phase in the presence of noise—and ambiguity—by generating a complete distribution of possible values for the desired parameters. By plotting, not simply traditional error bars, but instead all the bootstrap solutions, we can see the entire distribution of solutions in the absence of, or in the presence of, ambiguities. This is very useful and can be used for all measurements with FROG or any other method that involves more data points than are absolutely required to determine the desired pulse parameters. It will also be useful for finding new ambiguities (if they exist) and for finding "approximate ambiguities," that is, very different pulses that do not have identical traces, but traces that differ by less than the noise present in a particular measured trace. In short, the bootstrap method is a simple, powerful, and general method that is ideal for determining the uncertainty in ultrashort pulse measurement whether in the presence of ambiguities or not.